Numerical scheme for an efficient colour images denoising

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Abstract

In this paper, we are interested by the enhancement of colour images, where we present a numerical scheme to implement non linear diffusion filter. This scheme is developed to denoise colour images corrupted by additive noise. It is based on harmonic averaging that takes into account correlation between all colour components of the image. The proposed scheme is an efficient tool at image selective smoothing in presence of strong noise. This analysis shows that our method is more aggressive than some related works.

Keywords: Colour, diffusion, noise, gradient.

1. Introduction

The frequent problem in low-level vision arises from the wish to eliminate noise and uninteresting small scale details from the degraded image, without blurring semantically important structures in the image such as edges. Diffusion method has been proved to be an efficient filtering technique for simultaneously performing contrast enhancement and noise reduction. Several models have been proposed to enhance gray level images and some satisfactory results have been reported [1,2,3,4]. The extension of these methods to colour images can be achieved in two ways. The first one consists in using a marginal approach that enhances each colour component of the image separately using a scalar approach, that bases image processing on channel by channel filter. This technique is well known to produce false colours in the enhanced image, because each colour component is processed independently of the others. Indeed, any image is captured in given conditions that make colours interconnected referring to the real scene itself. So, enhancing any component without taking into account the existing dependence yields, in general case, to an undesired result. The second approach consists in using a single vector processing, where different components of the image are enhanced by considering correlation between them. The main idea here is to find a mathematical framework to relate different colour values and variations, while conserving the self behaviour of each component [5].

2. Diffusion filter

A simplest isotropic diffusion filter eliminates noise, but also blurs and dislocates important features in the image such as edges [6]. The image gets more and more simplified and the edges are dislocated when moving from the finer to coarser scales. Thus, both effects of blur and dislocation of edges can be avoided by inhibiting diffusion along edge direction while preserving an isotropic behaviour within stationary level gray regions. This can be made by a selective smoothing operation that results in distinguishing between intra- and inter-region smoothing [7,8], and returns the zones with weak gradient magnitude homogeneous and reduces the diffusion along edges in order to preserve and enhance them. Central to the technique is a diffusivity or edge stopping function that controls the degree of image smoothing over time. The anisotropic diffusion equation is given by:

$$u_t = \text{div}\left(g(s)\left|\nabla u\right|\nabla u\right)$$  \hspace{1cm} (1)

with $u(0)=u_0$ the initial image. $u_0$ is a smoothed version of the image using a Gaussian kernel with standard deviation $\sigma$. $g(s)$ is monotonically decreasing function in gradient magnitude and usually contains a free parameter, $K$, which determines the contrast of edges that will have significant effects on the smoothing. It is of the type:
\[ g(t) = \frac{1}{1 + \frac{t^2}{K^2}} \quad (2) \]

Smoothed versions of the initial image are obtained by choosing progressively greater values of \( t \) from the solution.

A simplest development of (1) [9] can yield to:
\[ u_t = g \left( \nabla u_x \right) u_{xx} + \left( \nabla g \left( \nabla u_x \right) \right) \nabla u + g \left( \nabla u_x \right) \nabla u \eta \eta \quad (3) \]

where \( u_{xx} \) and \( u_{\eta \eta} \) represent the diffusion terms in level set and gradient directions:
\[ u_{xx} = \frac{u_x u_x^2 - 2u_x u_y u_{xy} + u_y u_y^2}{u_x^2 + u_y^2} \quad (4) \]
\[ u_{\eta \eta} = \frac{u_x u_x^2 + 2u_x u_y u_{xy} + u_y u_y^2}{u_x^2 + u_y^2} \quad (5) \]

and
\[ \xi = \frac{1}{\sqrt{u_x^2 + u_y^2}} \left( -u_y \right) \quad (6) \]
\[ \eta = \frac{1}{\sqrt{u_x^2 + u_y^2}} \left( u_x \right) \quad (7) \]

So as a fundamental result, any diffusion method can be written as an addition of both kinds of diffusion:
\[ u_t = C_{\xi} u_{\xi \xi} + C_{\eta} u_{\eta \eta} \quad (8) \]

where \( C_{\xi} \) and \( C_{\eta} \) are control coefficients respectively in level set and gradient directions.

The extension of this concept to colour images could be efficiently achieved by modelling colour components dependence, in order to avoid the apparition of undesired structures in the processed image. As we can observe on figure 1, channel by channel diffusion filter has created false colours in the enhanced image.

Figure 1: Enhancement of Parrot image. (a) Original image; (b) Channel by channel nonlinear diffusion.

3. Colour images filtering

In order to describe colour image variations and structures, Di Zenzo proposed to use the local variation of a vector gradient norm \( |\nabla u| \) that detects edges and corners when its value becomes high [10]. It can be computed using the eigenvalues \( \lambda_+ \) and \( \lambda_- \) \((\lambda_+ > \lambda_-)\) corresponding to the eigenvectors respectively \( \theta_+ \) and \( \theta_- \) of the symmetric and semi-positive matrix \( G \):
\[ G = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} = \begin{pmatrix} u_x^2 + u_y^2 & u_x u_y \\ u_x u_y & u_y^2 \end{pmatrix} \]
\[ \lambda_+ = \frac{g_{11} + g_{22} \pm \sqrt{(g_{11} - g_{22})^2 - 4g_{12}^2}}{2} \quad (10) \]
\[ \theta_+ = \frac{1}{2} \arctan \frac{2g_{12}}{g_{11} - g_{22}} \quad (11) \]
\[ \theta_- = \theta_+ + \frac{\pi}{2} \quad (12) \]

Three different choices of vector gradient norms can be envisaged. \( \sqrt{\lambda_+} \), which is the value corresponding to the maximum variations. \( \sqrt{\lambda_- - \lambda_+} \) that fails to detect saddle points discontinuities. \( \sqrt{\lambda_+ + \lambda_-} \) that detects perfectly edges and corners. Hence using Di Zenzo idea, Sapiro and Ringach developed for colour images an extension of the mean curvature motion [11]. They exploit the following diffusion in level set direction \( \xi \) for gray level images [12]:
\[ u_t = g \left( |\nabla u_x| \right) u_{\xi \xi} \quad (13) \]

So, they proposed:
\[ u_p = g(\nabla u)|u_{p \theta \phi} \] (14)

and used the norm:

\[ |\nabla u| = \sqrt{\lambda_+ - \lambda_-} \] (15)

Once the new base \( \theta_+ = \left( \theta_1, \theta_2 \right) \) established, we can compute \( u_{p \alpha \beta \gamma} \) with a simple transformation:

\[ u_{p \alpha \beta \gamma} = \theta_1^2 u_{p \alpha \beta} + 2 \theta_1 \theta_2 u_{p \alpha \beta \gamma} + \theta_2^2 u_{p \gamma \gamma} \] (16)

Though all colour components of the image are smoothed along a common vector edge direction with a common intensity, undesired textures are created while edges are not perfectly enhanced. This is due to the fact that Sapiro-Ringach model uses one direction, which corresponds in the scalar case to level set direction. Diffusion in this direction is well known to produce analogue results in level gray images.

As a natural way, Tschumperlé and Deriche proposed to diffuse the image in both directions \( \theta_1 \) and \( \theta_2 \) in order to avoid as much as possible Sapiro-Ringach filter limitations. All components of the image evolve under the following model [13]:

\[ u_p = C_{\theta_1} u_{p \theta_1 \theta_1} + C_{\theta_2} u_{p \theta_2 \theta_1}, \] (17)

\( C_{\theta_1} \) and \( C_{\theta_2} \) are decreasing functions of the common gradient magnitude \( |\nabla u| \), which is computed by the norm:

\[ |\nabla u| = \sqrt{\lambda_+ + \lambda_-} \] (18)

\[ C_{\theta_1}(s) = \frac{1}{\sqrt{1+s^2}} \] (19)

\[ C_{\theta_2}(s) = \frac{1}{1+s^2} \] (20)

This filter smoothes well noise in homogeneous parts of the image, while it keeps edges enhanced referring to Sapiro-Ringach model. Feddern also diffuses the colour images using both directions \( \theta_1 \) and \( \theta_2 \) [14]. He bases his method on the following filter for gray level images [9,15]:

\[ u_p = |\nabla u| \text{div} \left( g \left( |\nabla u| \right) \frac{\nabla u}{|\nabla u|} \right) \] (21)

which can be written as:

\[ u_i = g \left( |\nabla u_i| \right) u_{i \theta \phi} \] (22)

So for colour images, Feddern chooses to replace the direction \( \vec{\xi} \) by \( \theta \), while he adds diffusion term in the second direction \( \theta_i \). And in order to more relate different components, Feddern computes the gradient magnitude using the norm (12). Feddern model is given by:

\[ u_p = \frac{1}{\rho} \left( \frac{\lambda_1 + \lambda_2}{2} \right) u_{p \theta \theta} + \frac{2 \lambda_1 \lambda_2}{\rho} u_{p \theta \gamma \gamma} + \frac{\lambda_1^2 + \lambda_2^2}{\rho} u_{p \gamma \gamma} \] (23)

This filter gives satisfactory results, in that it smoothes efficiently homogeneous parts of the image with edge conservation. In the other hand, Weickert has proposed to enhance colour components of the image under same and unique structure tensor, which has been used in order to take account of correlation between all components by constructing a coherence enhancing diffusion [16]:

\[ u_p = \text{div} \left( D \nabla u_p \right) \] (24)

\( D \) is a tensor that takes into account information from all components in the enhancement of the image. The diffusion tensor \( D \) is a positive definite matrix, which steers the diffusion process. Its eigenvalues \( \lambda_1 \) and \( \lambda_2 \) determine the diffusivities in the directions of the eigenvectors \( w_1 \) and \( w_2 \), so:

\[ \lambda_i = \alpha \] (25)

\[ \lambda_i = \alpha + (1-\alpha) \exp \left( -\frac{c}{(\mu_i - \mu_2)^2} \right) \text{ otherwise} \] (26)

with \( C > 0 \) and \( \alpha \in [0,1] \). \( \mu_1 \) and \( \mu_2 \) are the eigenvalues of the tensor structure \( J_\rho \), which posses the same eigenvectors \( w_1 \) and \( w_2 \) than the tensor \( D \). \( J_\rho \) is given by:

\[ J_\rho = \sum_{p=1}^{3} G_\rho \ast \nabla u_{p \alpha} \nabla u_{p \alpha \alpha} \] (27)

where \( G_\rho \) is the Gaussian function with standard deviation \( \rho \) and \( \ast \) denotes the convolution operator.

This filter smoothes well noise in homogeneous parts of the image and enhances edges, while it suffers from an excessive oriented diffusion that creates a directional objects in the restored image.
4. Proposed scheme

The proposed method is based on self-snakes model (21), that introduces an edge-stopping function into mean curvature motion in order to prevent further reduction of the level lines once they have reached important image edges [9,15]. So, to expand this model to colour images, we propose a numerical scheme that takes account of correlation between all colour components of the image. The continuous form is given by:

\[
    u_{p,t} = \|\nabla u\| \text{div} \left( g \left( \left| \nabla u \right| \right) \frac{\nabla u_{p}}{\|\nabla u\|} \right)
\]

(28)

We notice that outside the term div(\cdot), we employ a common gradient magnitude \( \|\nabla u\| \) that can be computed by the norm (12). Inside div(\cdot), we use \( \nabla u_p \) in the numerator in order to introduce the self behaviour of each colour component, while we conserve the common norm in the denominator and the smoothed image. This choice excludes the creation of new structures in the enhanced image. That observation can be well done in the excessive smoothing of Flower image (figure 2), where false colours appear in the result of channel by channel method:

\[
    u_{p,t} = \|\nabla u_{p}\| \text{div} \left( g \left( \left| \nabla u_{p} \right| \right) \frac{\nabla u_{p}}{\|\nabla u\|} \right)
\]

(29)

New colours have been created like blue and red, which don’t exist in the original image. In our case, any new colour doesn’t have been introduced.

However, one can choose greater value of the step time \( \tau \) in the numerical scheme, in order to converge to the study state quickly with few iterations. Because as it is mentioned in [17], such a scheme is absolutely stable. Figure 3 shows an experiment on a bad JPEG compressed image, where the restored image has been obtained after one iteration only inhibiting JPEG effect.

![Figure 2: Excessive smoothing of Flower image. (a) Original image; (b) Channel by channel method (29); (c) Proposed method.](image)

![Figure 3: Inhibiting of JPEG effect. (a) Compressed image; (b) Solution with one iteration and \( \tau = 500 \). (c) Enlargement of (a); (d) Enlargement of (b).](image)
positive integer and τ is the time step size. By $u_{p}^{k}$, $\nabla u_{p}^{k}$, $|\nabla u_{p}^{k}|$, we denote approximations of $u(x_{i},t_{k})$, $\nabla u_{p}$, $|\nabla u_{p}|$, $g(|\nabla u_{p}|)$ respectively. For the implementation of the temporal derivative $u_{p}$, respecting to the component $p$, we use:

$$u_{p_{i}} = \frac{u_{p_{i}}^{k+1} - u_{p_{i}}^{k}}{\tau}$$  \hspace{1cm} (30)$$

For the to approximate of the quasi-divergence term, we employ the semi-implicit scheme with harmonic averaging [17,18]:

$$\nabla u_{p}^{k} \cdot \nabla u_{p}^{k} = \nabla u_{p}^{k} \sum_{j \in w(i)} \frac{2}{\nabla u_{p}^{k} \times \nabla u_{p}^{k}}$$  \hspace{1cm} (31)$$

where $w(i)$ consists on the four neighbours of pixel $i$ with $j \in w(i)$. And

$$|\nabla u_{p}^{k}| = \sqrt{\sum_{i \in \{x,y\}} (|\nabla u_{p}^{k}|)^{2}}$$  \hspace{1cm} (32)$$

Therefore, the numerical implementation of the proposed model can be written as:

$$\frac{u_{p_{i}}^{k+1} - u_{p_{i}}^{k}}{\tau} = |\nabla u_{p}^{k} \sum_{j \in w(i)} \frac{2}{\nabla u_{p}^{k} \times \nabla u_{p}^{k}}$$  \hspace{1cm} (33)$$

Thus (34),

$$u_{p_{i}}^{k+1} = u_{p_{i}}^{k} + \frac{2}{h^{2}} \sum_{j \in w(i)} \left[ \left| \nabla u_{p}^{k} \right| \left( \nabla u_{p}^{k} \times \nabla u_{p}^{k} \right) \right]$$  \hspace{1cm} (34)$$

So by using matrix-vector representation [8,17,18,19], we obtain:

$$u_{p_{i}}^{k+1} = u_{p_{i}}^{k} + \tau A(u_{i}) u_{p_{i}}^{k+1}$$  \hspace{1cm} (35)$$

where components of the matrix $A$ are given by:

$$A_{ij} = \begin{cases} 2 & j \neq i, j \in w(i) \\ \frac{2}{h^{2}} \sum_{j \in w(i)} \left[ \left| \nabla u_{p}^{k} \right| \left( \nabla u_{p}^{k} \times \nabla u_{p}^{k} \right) \right] & j = i \\ 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (36)$$

w(i) represents the two neighbouring pixels with respect to the direction $l \in \{x,y\}$. Boundary pixels may have only one neighbour.

Hence by decomposing the matrix $A$ in $x$ and $y$ directions, we yield to:

$$\left( I - \tau \sum_{l \in \{x,y\}} A_{l} \right) u_{p_{i}}^{k+1} = u_{p_{i}}^{k}$$  \hspace{1cm} (37)$$

However, the solution $u_{p_{i}}^{k+1}$ cannot be directly determined. It requires solving a linear system of equations. Its solution is formally given by:

$$u_{p_{i}}^{k+1} = \left( I - \tau \sum_{l \in \{x,y\}} A_{l} \right)^{-1} u_{p_{i}}^{k}$$  \hspace{1cm} (38)$$

5. Comparison

In this section, we evaluate performances of our model referring to filters of Sapiro-Ringach, Weickert, Tschumperlé-Deriche and Feddern. We compare qualitatively and quantitatively the results by considering highly degraded images. We choose the parameters that give better results for each method. All filters are applied to noised images that we generate by adding random Gaussian noise to the original image. So, the first experiment is on Synthetic image with coloured geometrical forms (figure 4). Degraded image is corrupted with a strong noise (SNR=0 dB). We can see how our model has efficiently removed the strong noise in smoother regions with edges preserving referring to other models. Coloured forms appear smooth and well contrasted. Only Feddern model gives satisfactory results, in that it denoises homogeneous regions, while it doesn’t conserve some edges. In the case of Sapiro-Ringach and Tschumperlé-Deriche models, edges are lost and in the results of Weickert method, oriented textures have been introduced in the restored image. In figure 5, our SNR value is always greater than the SNR of other methods. The second
experiment is on Cartoon image with SNR=4.65 dB (figure 6). The image has been well denoised by all methods. In the case of Sapiro, some edges have been smoothed, while in the result of Weickert, like in the first experiment, oriented textures have been created. The enhanced image by Tschumperlé-Deriche model has a smooth appearance. However, Feddern filter conserves well edges referring to other methods. Only our model conserves better edges with a good colour contrast. As we can see on the background, our method has highlighted levels of yellow and orange colours that appear in the original Cartoon image. We remark that our SNR is better referring to all methods (figure 7). On Flower image with SNR=5 dB (figure 8), we notice here again that our model gives the best result at denoising of homogeneous parts of the image like the flower, sheets and background with edges enhancement, without creating any false colours. Only Feddern model denoised well the image with a smooth aspect of edges. Our SNR is again better than the other SNRs (figure 9).

6. Conclusion

We have proposed a numerical scheme to implement anisotropic diffusion based on main curvature motion. The idea behind this scheme is to use two gradient values in order to take account of correlation between all components of the colour image, while we conserve the self behaviour of each component in the enhancement of the image. This analysis shows that our model is more efficient than some related methods in restoration of colour images in presence of strong noise, in that it smoothes efficiently homogeneous parts of the image, while it keeps edges enhanced without creating false colours.

References


Figure 4: Enhancement of Synthetic image. (a) Noisy image; (b) Sapiro-Ringach model with $\sigma=1$, $K=18$, $\tau=0.2$ and 1500 iterations; (c) Weickert model with $\sigma=1$, $\rho=4$, $c=1$, $\alpha=0.0000001$, $\tau=0.2$ and 1500 iterations; (d) Tschumperlé-Deriche model with adaptive $\tau=15$ and 1500 iterations; (e) Feddern model with $\sigma=1$, $K=5$, $\tau=0.2$ and 1500 iterations; (f) Proposed model with $\sigma=1$, $K=10$, $\tau=0.2$ and 1500 iterations.

Figure 5. SNR representation of Synthetic image as a function of the number of iterations.
Figure 6: Enhancement of Cartoon image. (a) Noisy image; (b) Sapiro-Ringach model with $\sigma=1$, $K=20$, $\tau=0.2$ and 200 iterations; (c) Weickert model with $\sigma=1$, $\rho=4$, $c=3$, $\alpha=0.00001$, $\tau=0.2$ and 200 iterations; (d) Tschumperlé-Deriche model with adaptive $\tau=10$ and 200 iterations; (e) Feddern model with $\sigma=1$, $K=3.5$, $\tau=0.2$ and 200 iterations; (f) Proposed model with $\sigma=1$, $K=3$, $\tau=0.2$ and 200 iterations.

Figure 7. SNR representation of Cartoon image as a function of the number of iterations.
Figure 8: Enhancement of Flower image. (a) Noisy image; (b) Sapiro-Ringach model with $\sigma=1$, $K=12$, $\tau=0.2$ and 100 iterations; (c) Weickert model with $\sigma=1$, $\rho=4$, $c=1$, $\alpha=0.001$, $\tau=0.2$ and 100 iterations; (d) Tschumperlé-Deriche model with adaptive $\tau=15$ and 100 iterations; (e) Feddern model with $\sigma=1$, $K=5$, $\tau=0.2$ and 100 iterations; (f) Proposed model with $\sigma=1$, $K=3$, $\tau=0.2$ and 100 iterations.

Figure 9. SNR representation of Flower image as a function of the number of iterations.