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Wave optics : Courses and exercises

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Introduction to Wave Optics: Courses and exercises

While the ray model of geometric optics provides a powerful and intuitive framework for understanding phenomena like reflection, refraction, and basic image formation in lenses and mirrors, it represents an approximation that ultimately fails to capture the full, rich behavior of light. Many crucial optical effects can only be understood by embracing light's fundamental nature as an electromagnetic wave. This is the domain of Wave Optics, also known as Physical Optics. Rooted in Maxwell's comprehensive electromagnetic theory, experimentally verified by Hertz, wave optics delves into phenomena that arise directly from the superposition, bending, and vector nature of light waves. Crucially, wave optics explains effects that are inexplicable within the ray approximation, such as interference (the superposition of coherent waves), diffraction (the bending of waves around obstacles), and polarization (the orientation of wave oscillations).

Beyond its fundamental importance in describing the physical world, wave optics is not merely an academic subject; it underpins countless technologies and natural phenomena that shape our daily lives. We encounter wave optics constantly: the iridescent shimmer of colors on a soap bubble or an oil slick arises from thin-film interference; the operation of CDs, DVDs, and Blu-ray discs relies on diffraction gratings; Polaroid sunglasses utilize polarization to eliminate reflected glare; and anti-reflection coatings on camera lenses and eyeglasses are meticulously designed using interference principles.

In communications, the modern information age is built on fiber optics. Understanding wave propagation, dispersion, modal properties, and interference effects within these fibers is critical for transmitting vast amounts of data at high speeds. Specialized optical filters based on interference (like Fiber Bragg Gratings or thin-film filters) are essential components in wavelength division multiplexing (WDM) systems that dramatically increase network capacity.

Wave optics is also indispensable in medicine for diagnostics and treatment. Optical Coherence Tomography (OCT), providing cross-sectional views of tissues like the retina, operates on low-coherence interferometry. High-resolution microscopy is fundamentally limited by diffraction, driving the development of advanced imaging techniques. Fiber optics enable minimally invasive endoscopy, guiding light into and images out of the body.

Furthermore, wave optics drives innovation across science and technology. Precision measurements in metrology often rely on interferometers capable of detecting sub-wavelength displacements. Diffraction gratings are core components of spectrometers used to analyze chemical compositions. Material science utilizes diffraction principles to determine crystal structures. Polarization is key to LCD technology and specialized optical sensors, while holography offers unique possibilities for 3D imaging and data storage.

To understand and harness these diverse applications, a solid grasp of the fundamental principles is paramount. This handout provides a structured approach to wave optics, designed for Master's level students in Physics and Material Sciences. It is organized into four main chapters: Light Waves, Polarization, Interference, and Diffraction. Through the theoretical explanations ("Courses") and carefully selected ("Exercises") presented herein, this resource aims to foster a deep and lasting understanding of wave optics, equipping students with the foundational knowledge necessary for advanced studies, research, and application in various scientific and technological fields.

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Introduction to Light Waves

In this chapter, we explore the fundamental nature of light as an electromagnetic wave. Unlike geometric optics, which treats light as rays, wave optics describes light in terms of propagating waves governed by Maxwell's equations. This perspective is essential for understanding phenomena such as interference, diffraction, and polarization, which cannot be explained using ray optics alone.

We begin by reviewing the historical development of light wave theory, from early corpuscular models to the modern electromagnetic description. Next, we derive the wave equation from Maxwell's laws and examine key properties of light waves, including phase, wavelength, and propagation in different media. Finally, we introduce complex notation for wave representation and discuss practical applications of these concepts in optics and photonics.

I.1. Historical Models of Light:

Understanding the Nature of Light?

The question of light's fundamental nature has intrigued scientists for centuries, leading to one of the most fascinating debates in physics history. Early inquiries often wrestled with two primary competing ideas:

I.1.1. The Corpuscular Model:

Championed most notably by **Sir Isaac Newton** in the 17th century, this model viewed light as composed of tiny particles ("corpuscles") traveling in straight lines. This perspective successfully explained phenomena like reflection (particles bouncing off surfaces) and refraction (particles changing speed and direction when entering a new medium). Newton's immense authority lent significant weight to this theory for over a century. While not entirely accurate for diffraction as we understand it now, proponents attempted to explain some bending effects through forces acting on the corpuscles near edges. The modern **photon model**, introduced by **Albert Einstein** in 1905 to explain the photoelectric effect, revived the particle aspect in a quantum mechanical context, where light consists of discrete energy packets (photons) with energy $E = h\nu$ (h being Planck's constant, ν the frequency).

I.1.2. The Wave Model:

Proposed around the same time as Newton's theory by **Christiaan Huygens**, this model suggested light propagates as waves spreading through a medium (initially hypothesized as the "luminiferous ether"). Huygens' principle, stating that every point on a wavefront acts as a source of secondary wavelets, could also explain reflection and refraction. However, conclusive experimental evidence for uniquely wave-like phenomena like interference and diffraction was initially lacking, and the model struggled to gain broad acceptance against Newton's influence.

I.1.3. The Turning Point:

The wave model gained significant ground in the early 19th century. **Thomas Young's** famous double-slit experiment (circa 1801) demonstrated **interference** – the characteristic pattern of bright and dark fringes formed when light from two coherent sources overlaps – providing compelling evidence for light's wave nature. Shortly after, **Augustin-Jean Fresnel** provided a rigorous mathematical framework for the wave theory, successfully explaining **diffraction** (the bending of light around obstacles) and discovering phenomena related to **polarization**, further solidifying the wave interpretation.

I.1.4. Electromagnetic Synthesis: In the 1860s, **James Clerk Maxwell** achieved a monumental unification, showing theoretically that light is a high-frequency **electromagnetic wave**, propagating at a specific speed 'c' in vacuum. His theory predicted that these waves consist of oscillating electric and magnetic fields perpendicular to each other and the direction of propagation. **Heinrich Hertz's** experimental generation and detection of electromagnetic waves in the 1880s provided stunning confirmation of Maxwell's theory.

Wave-Particle Duality: The 20th century brought quantum mechanics, revealing that light exhibits both wave-like and particle-like properties, a concept known as **wave-particle duality**. The appropriate model depends on the phenomenon being studied.

Which Model to Use?

- When the dimensions of obstacles or apertures are much larger than the wavelength of light ($D >> \lambda$), the simplified **geometric optics** model (treating light as rays) is often sufficient for analyzing image formation by lenses and mirrors.

- However, to understand phenomena like **interference, diffraction, and polarization**, which are central to this course, the **wave model** is indispensable. We must consider light's propagation as waves, often described by Maxwell's electromagnetic theory. The modern photon concept is crucial for light-matter interactions (like the photoelectric effect or emission/absorption), but the classical wave model remains the essential tool for explaining how light behaves as it propagates and interacts with structures comparable in size to its wavelength (where $D \sim \lambda$).

I.2. Wave aspect of light (electromagnetic wave)

1.2.1. Maxwell's Equations and Wave Equation:

Maxwell's electromagnetic theory completed the wave theory by introducing two vector quantities which are the vibrating quantities of the luminous phenomenon: light appears, in the case of a monochromatic wave, as consisting of an electric field and a magnetic field varying sinusoidally with time. By adopting the approach followed by J.C. Maxwell, we can affirm that light is an electromagnetic wave characterized by the association of a field electric field $\vec{E}$, a magnetic field $\vec{B}$ and a wave vector $\vec{k}$. This statement obviously needs to be verified.

In a vacuum, the $\vec{E}$ and $\vec{B}$ fields that define the electromagnetic wave must satisfy Maxwell's (1860) four equations:

$$div\,\vec{E} = 0$$

$$div\,\vec{B} = 0$$

$$\overrightarrow{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\overrightarrow{rot}\,\vec{B} = \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}$$

When combined, these equations lead to the equations of electric or magnetic field propagation can be written as:

$$\overrightarrow{rot}((\overrightarrow{rot}(\vec{E})) = -\frac{\partial \overrightarrow{rot}\,\vec{B}}{\partial t} \Rightarrow \Delta\vec{E} - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \vec{0}$$

$$\overrightarrow{rot}((\overrightarrow{rot}(\vec{B})) = \varepsilon_0\mu_0 \frac{\partial \overrightarrow{rot}\,\vec{E}}{\partial t} \Rightarrow \Delta\vec{B} - \varepsilon_0\mu_0 \frac{\partial^2 \vec{B}}{\partial t^2} = \vec{0}$$

These equations are similar to the wave equation:

$$\Delta u - \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} = 0 \quad with \;\; u = E \; or \; B$$

And $v = \sqrt{\frac{1}{\mu_0\varepsilon_0}}$ is the speed of propagation (depends on the nature of the medium). When Maxwell calculated this value using the experimentally known values of μ_0 and ε_0 (determined from electric and magnetic experiments), he found it was astonishingly close to the measured speed of light! This was compelling evidence that light *is* an electromagnetic wave. This speed in vacuum is a universal constant, denoted by **c**: c ≈ 2.998 x 10^8 m/s.

- ***The great merit of Maxwell's equations is that their combination leads to the propagation equations.***

I.2.2. Propagation in a Medium

- When light travels through a dielectric material (like glass or water), it interacts with the material's atoms. This slows down the wave propagation. The speed 'v' in the medium is related to the speed in vacuum 'c' by the refractive index 'n' of the medium: v = c/n (where n > 1).
- The fundamental constants ε_0 and μ_0 are replaced by the material's permittivity ε and permeability μ. So, v = 1 / √(με). Since n = c/v, we get n = √(με) / √($\mu_0\varepsilon_0$) = √(μr εr), where μr and εr are the relative permeability and permittivity.

I.2.3. Vector model of light

The **vector model of light** emphasizes that light, as an electromagnetic wave, is fundamentally described by the vector quantities of the electric field ($\vec{E}$), the magnetic field ($\vec{B}$), and the direction of propagation, represented by the wave vector ($\vec{k}$). Crucially, these three vectors maintain a specific geometric relationship dictated by Maxwell's equations: $\vec{E}$ and $\vec{B}$ are always mutually perpendicular, and both are also perpendicular to the direction of wave travel $\vec{k}$. This confirms the **transverse nature** of light waves, where the field oscillations occur in a plane orthogonal to the energy flow. Furthermore, $\vec{E}$, $\vec{B}$, and $\vec{k}$ form a **direct trihedral**.

The specific spatial arrangement of these fields depends on the source, leading to distinct wave shapes that are solutions to the wave equation; the most fundamental forms include **spherical waves**, which emanate radially from a point source with expanding spherical wavefronts, and **plane waves**, characterized by flat, parallel wavefronts and a constant wave vector $\vec{k}$ (figure I.1), often used to approximate light from distant sources or laser beams. This vector description is essential for understanding phenomena like polarization and energy transport, providing the physical underpinning for simpler scalar treatments.

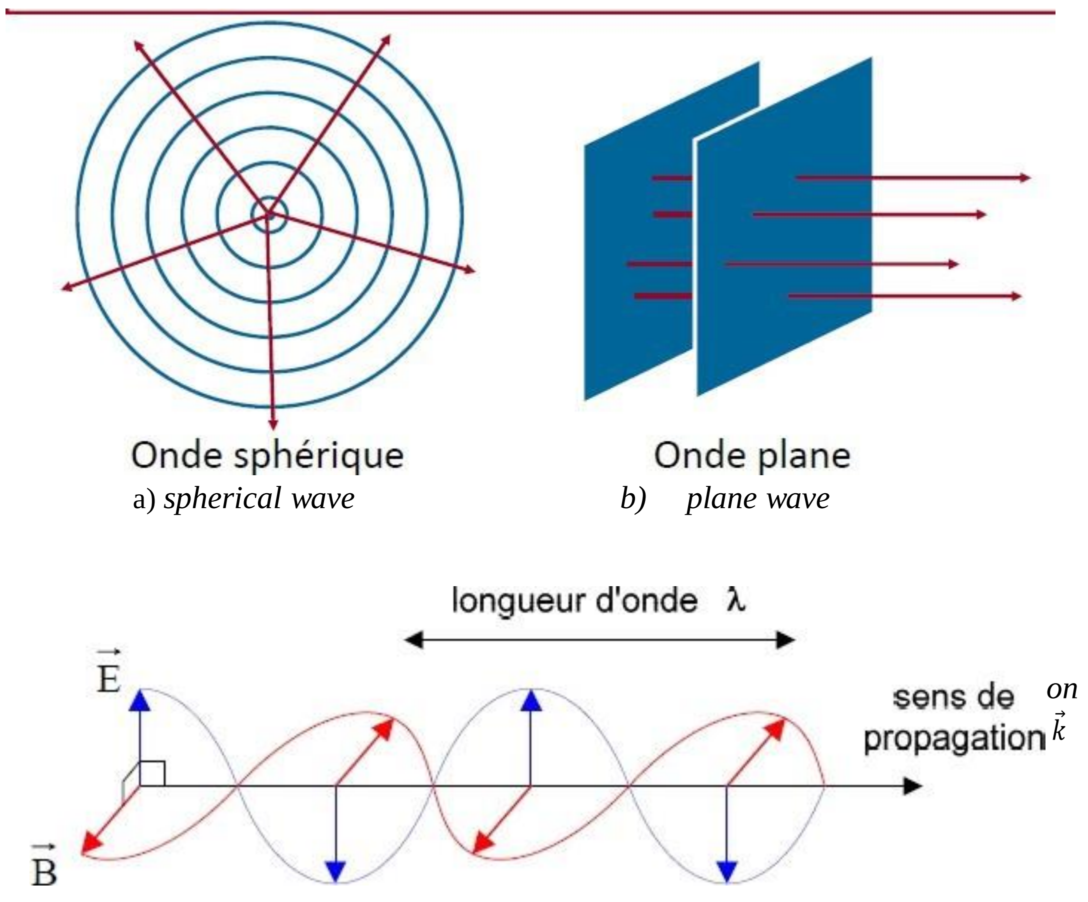


Figure I.1: S*pherical wave shape(a) , Plane wave shape (b)*, direct trihedral vectors $\vec{E}, \vec{B}, \vec{k}$ representation (c)

I.2.4. Importance of the Vector Model:

- It provides the accurate physical picture underlying the simpler scalar models often used later.
- It is absolutely essential for understanding **polarization** (Chapter II), which deals specifically with the orientation of the **E** vector in the plane perpendicular to **k**.
- It's needed for calculating energy flow using the Poynting vector.
- It connects the abstract mathematical solutions of Maxwell's equations to concrete physical wave geometries (plane and spherical).

I.3. Wave Characteristics:

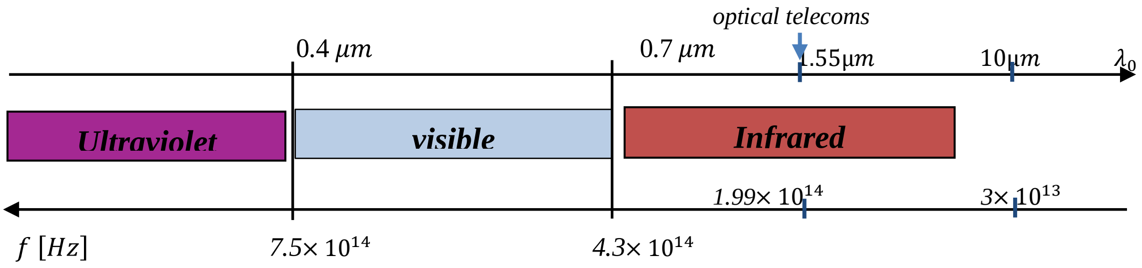
Fourier analysis allows us to consider the wave *A*(*M*,*t*) emitted by any source, even a complex one, as a sum of simple sinusoidal functions of time, each with a specific angular frequency or pulsation **(*ω*).** Focusing on these fundamental sinusoidal components, we find that, like all waves, electromagnetic waves possess a **dual periodicity**: the periodicity of the phenomenon in **space** is measured by the **wavelength (*λ*)** (in meters), while the periodicity in **time** is measured by the **period (*T*)** (in seconds) or its inverse, the **frequency (*f* = 1/*T*)** (in Hertz), which our eyes perceive as color.

The light ⟺ ***Electromagnetic wave***

- When light propagates through a transparent material medium, its interaction with the atoms slows it down to a ***speed :*** $\boldsymbol{v = \frac{c}{n}}$
- In the vacuum of space, light travels at a maximum and universal speed, denoted by $cc \approx 3.10^8 (\frac{m}{s})$

  where ***n*** is the **refractive index** of the medium (n ≥ 1).

- ***The frequency f corresponds to the color of light***



A key principle is that the **frequency (*f*)**, determined by the source, **remains constant** as light transitions between media. Therefore, since the speed (v) changes while the frequency (*f*) does not, the **wavelength must adapt** accordingly

$$\lambda = v\mathrm{T} \qquad\qquad v = \frac{c}{n}$$

$$\lambda = \frac{c}{n}T$$

Defining the vacuum wavelength as $\lambda_0 = cT$, the wavelength within the medium is given by $\lambda = \frac{\lambda_0}{n}$

I.4. Plane wave in vacuum

I.4.1. Monochromatic traveling plane wave

The light wave is said to be plane if the wave vector $\vec{k}$ which dictates the direction of propagation, is constant in both direction and sense throughout space. Consequently, the surfaces of constant phase (wave fronts) are infinite planes (containing vectors $\vec{E}$ and $\vec{B}$) perpendicular to $\vec{k}$. "**Monochromatic**" implies the wave consists of a single, well-defined frequency (***f***) or angular frequency (***ω*** **= 2*πf***), meaning its color is pure (thewave vector is of constant modulus). "**Harmonic**" indicates that the wave's oscillation (e.g., the electric field variation) follows a sinusoidal pattern in both space and time. Finally, "**Progressive**" means the wave propagates, carrying energy through space over time, rather than being a standing wave. For such a wave traveling along the x-direction with speed v, its general mathematical expression, often representing a component of the electric or magnetic field, is given by:

$$\boldsymbol{S(x,t) = A_M \cos\left[\omega\left(t - \frac{x}{V}\right) + \varphi_0\right]}$$

$\boldsymbol{A_M}$: Maximum amplitude

$\boldsymbol{\omega}$: The angular frequency (pulsation)

$\boldsymbol{\varphi_0}$: The initial phase at the origin

$\varphi\,(x,t)$: $\boldsymbol{\omega\left(t - \frac{x}{V}\right) + \varphi_0}$ Phase of the monochromatic wave

The function $\boldsymbol{S(x,t)}$ represents the electric or magnetic field. This mathematical form is a fundamental solution to the wave equation and serves as a cornerstone for analyzing interference and diffraction.

I.4.2. Surfaces of constant phase or wave surface

For a plane wave, equal phase surfaces are planes. For the plane wave, the equal phage planes are orthogonal to the direction of propagation (figure I.2).

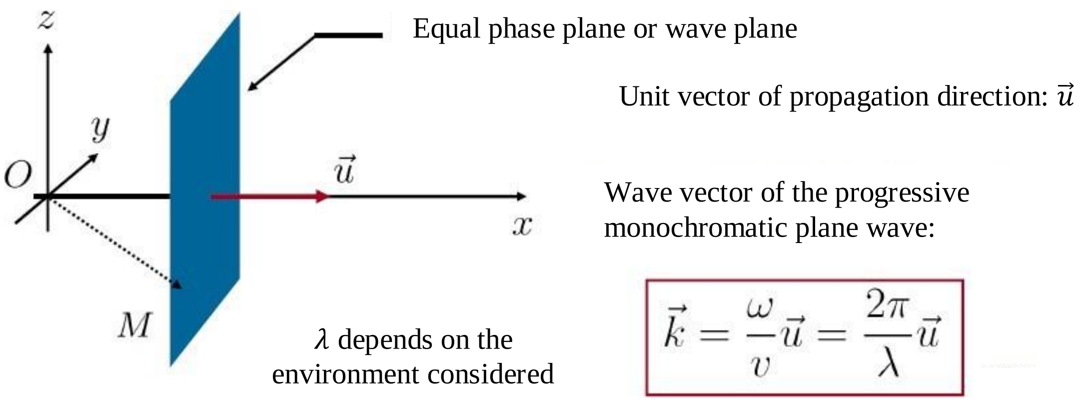


Figure I.2: Equal Phase plane representation

I.4.3. General case

we write the phase $\varphi(\vec{r},t) = \omega t - \vec{k}.\vec{r} + \varphi_0$

Where $\boldsymbol{r}$ is the vector marking the current point $\boldsymbol{M}$ and is written as:

$$\vec{r} = x.\vec{u}_x + y.\vec{u}_y + z.\vec{u}_z = \overrightarrow{OM}$$

choosing: $\vec{k} = \frac{\omega}{v}\vec{u}_x$

You'll find: $\varphi(\vec{r},t) = \omega.\left(t - \frac{x}{V}\right) + \varphi_0$

I.5. Complex notation

It is possible to add an imaginary part to the real field, to obtain a complex field:

$$S(x,t) = A_M e^{i\theta(x,t)}$$

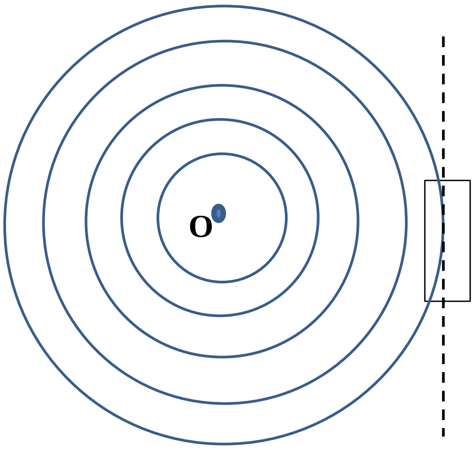
The real field can be found using the relation

$$S(x,t) = \frac{1}{2}[S(x,t) + S^*(x,t)] = Re[S(x,t)]$$

I.6. Monochromatic progressive spherical wave

- For spherical waves, the wave vector is radial.
- Wave surfaces are spheres

$$\boldsymbol{A(r,t) = \frac{A_0}{r} e^{i(\omega t - \frac{2\pi}{\lambda} r + \varphi_0)}}$$



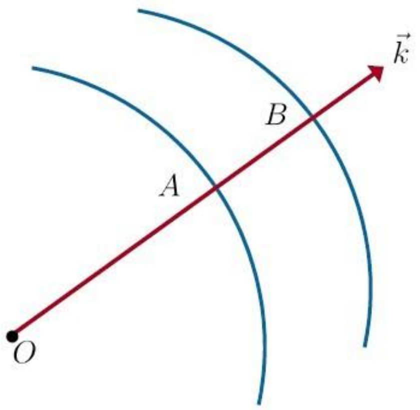
→ Far from the source, we can locally assimilate the spherical wave to a plane wave

Condition of validity: $r \gg \lambda$

Figure I.3: assimilation of a plane wave to a spherical wave

I.7. Phase difference between two points on the same light beam

In a homogeneous medium of index n, a rectilinear light ray is determined by anypoint O and its unit vector $\vec{u}$. A being any point on this ray $\vec{r_A} = \vec{u}.OA$ is the length traveled by the light. between A and B, counted positively in the direction of propagation, the phase difference of the wave can be written as:



$$\varphi(\vec{r}_A) = \omega t - \vec{k}\cdot\overrightarrow{OA} + \varphi_0$$
$$\varphi(\vec{r}_B) = \omega t - \vec{k}\cdot\overrightarrow{OB} + \varphi_0$$

At time t: the phase acquired by the wave between points A and B is

$$\varphi_{AB} = \varphi(\vec{r}_B) - \varphi(\vec{r}_A) = \frac{2\pi n}{\lambda_0}\vec{u}\cdot\left(\overrightarrow{OA} - \overrightarrow{OB}\right)$$

$$\varphi_{AB} = -\frac{2\pi n}{\lambda_0}\overline{AB}$$

λ_0 : wavelength in vacuum

Either: $\varphi_{AB} = -\frac{2\pi}{\lambda_0} L_{AB}$

I.8. Introducing the monochromatic plane wave

Structure of monochromatic plane electromagnetic wave :

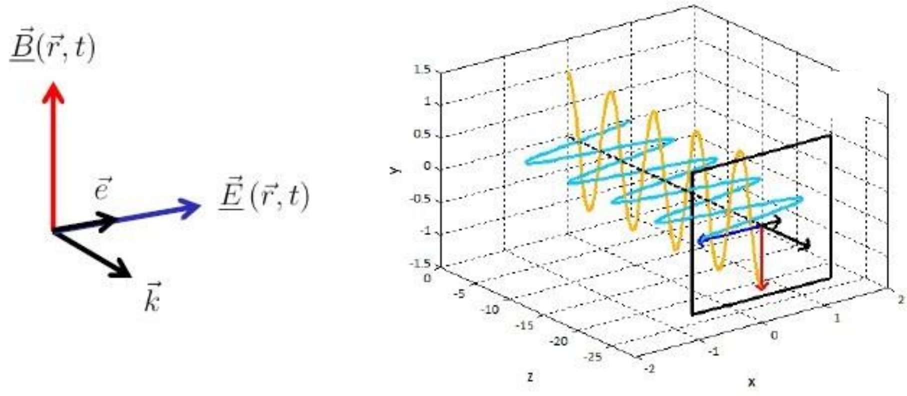


Figure I.4: electromagnetic wave representation

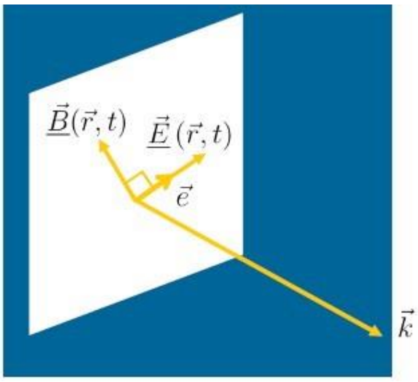
In the chosen example in figure I.4.:

- Direction of propagation: (or k//Oz)
- direction of polarization of the (electric field):Ox
- wave plane: parallel to (x,y)

The monochromatic progressive plane electromagnetic wave polarized rectilinearly (case of the isotropic medium):

$$\underline{\vec{E}}(\vec{r},t) = E_0 e^{j\left(\omega t - \vec{k}\cdot\vec{r}\right)}\vec{e}$$

$$\underline{\vec{B}}(\vec{r},t) = \frac{\vec{k}}{\omega} \wedge \underline{\vec{E}}(\vec{r},t)$$



- Light is a transverse wave: the electric and magnetic fields are in a plane orthogonal to the direction of propagation.
- $\vec{e}$ is the unit vector of the direction carrying the electric field. we call this direction the direction of polarization

I.9. Pointing vector

The direction of propagation of electromagnetic energy is given by the Pointing vector:

$$\vec{P} = \frac{\vec{E} \wedge \vec{B}}{\mu_0}$$

The average of the norm of this vector is the power per unit area I (W.m^{-2}):

$$I = \langle \|\vec{P}\| \rangle = \frac{1}{2}\varepsilon_0 c E_0^2$$

Consequence $\vec{P}$ is orthogonal to $\vec{E}$ and to $\vec{B}$, therefore parallel to $\vec{k}$ ⇒The energy propagates in the same direction as the wave vector $\vec{k}$

I.10. Electromagnetic wave representation and scalar theory of light

In general we only represent the electric field $\vec{E}$ because:

- The magnetic field $\vec{B}$ is known if we know $\vec{E}$ (thanks to Maxwell'sequations).
- The eye, like optical sensors, is sensitive to $\vec{E}$ and not $\vec{B}$. (Aux At current light intensities, the effect of the EM wave's electric field on the charges making up the material is dominant over the effects of the magnetic field (this is n o longer true if the intensity is greater than ...10^{18} W.cm^{-2} !!!).⟹ At this stage, a single vector field is all that's needed to describethe light vibration.
- The electromagnetic wave is transverse and, for simplicity's sake, flat.
- In most of the interference and diffraction experiments we're going to study, the "light rays" (=poynting vector or light intensity) that interfere will be almost parallel at a given point M, so the different electric fields (corresponding to the different rays) will all be practically contained in the same plane perpendicular to the direction of propagation ⟹ they can always be decomposed along common axes(for two waves with the same rectilinear polarization, the projection axis is unique).
- Light vibration will therefore be considered as a scalar quantity, the projection on a common axis of the electric field vector: this quantity will be denoted $S(M,t)$.

I.11. Wave train

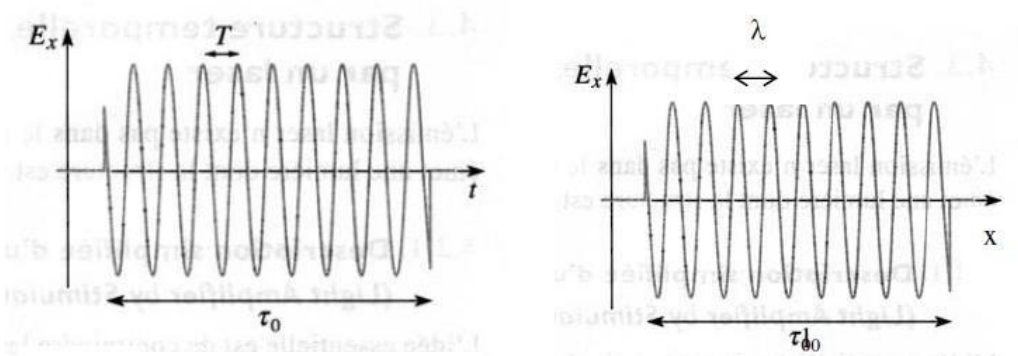
A monochromatic traveling plane wave with direction of propagation (oz) is of the form :

$$\vec{E}(M.T) = E_{Mx}\cos(\omega t - k.z + \varphi_1)\vec{u}_x + E_{My}\cos(\omega t - k.z + \varphi_2)\vec{u}_y$$

For t between -∞ and -∞ and whatever z. An individual atom emits for a very short time, of the order of $\tau_0 = 10^{-11} s$ (called coherence time). ⟹ At a given point the wave train reception time is τ_0 (Figure I.5).

At a given instant, the length of the wave train is $l_0 = c\tau_0$ (called the temporal coherence length, of the order of a few mm) (Figure I.5).

❖ Note that c is the speed of light in a vacuum. In the case of a material medium where the speed is v, we would write: $l_0 = v\tau_0$



Wave train: E(x) at a fixed point

Wave trains: E(x) at a given time as a function of x

Figure I.5: The coherence time and the length of the wave train

The light wave received by a point M is the sum of the wave trains emitted by a large number of atoms. Its structure depends on the nature of the source (classical or laser). The wave emitted by a classical (natural) source is an uninterrupted succession of wave trains of duration around τ_0 and incoherent. In other words, there is no phase relationship between them. The source is said to be incoherent.

A laser source has a much greater coherence time of the order $\tau_0 = 10^{-7} s$ and a corresponding coherence length of the order of several meters: such a source is said to be coherent.

I.12 Conceptual Questions and Exercises for Chapter I

1. **Calculation:** A green laser pointer emits light with a vacuum wavelength (λ_0) of 532 nm.
   - What is its frequency (f)?
   - What is its angular frequency (ω)?
   - If this light enters water ($n \approx 1.33$), what are its speed (v), frequency (f), and wavelength (λ) in the water? (Relates to p.7-8)
2. **Interpretation:** A plane wave in vacuum is described by the electric field $E_y(z, t) = E_0 \cos(\omega t - kz)$.
   - What is the direction of propagation?
   - What is the direction of polarization?
   - Write down the expression for the corresponding magnetic field $\vec{\boldsymbol{B}}$. (Relates to p.11-12)
3. **Concept:** What is meant by the "coherence length" of a light source? Why is a laser typically much more coherent than a light bulb? (Relates to p.13-14)
4. **Concept:** Under what conditions can we approximate a spherical wave as a plane wave? (Relates to p.10, Fig I.3)
5. **Calculation:** Calculate the wave number (k) in vacuum for the green laser light in Q1.
6. **Calculation:** Two points, A and B, are separated by 1.0 m along the direction of propagation of the green laser light (Q1) in vacuum. What is the phase difference ($\Delta\varphi$) between the waves at A and B? (Relates to p.10-11)
7. **Concept:** Why is it often sufficient to use a scalar wave S(M, t) to describe light in interference and diffraction experiments, instead of the full vector fields **E** and **B**? (Relates to p.12-13)
8. **Conceptual:** Explain why light is described as a transverse electromagnetic wave. What does this imply about the relative directions of the electric field (**E**), magnetic field (**B**), and the direction of wave propagation (**k**)? (Relates to Fig I.1c, I.4, p.12)
9. **Wave Properties in Media:** A helium-neon laser emits red light with a vacuum wavelength (λ_0) of 632.8 nm.
   (a) Calculate the frequency (f) and angular frequency (ω) of this light in vacuum. (b) The light beam enters a glass block with a refractive index (n) of 1.50. Calculate the speed of light (v), the frequency (f), and the wavelength (λ) of the light inside the glass.
10. **Plane Wave Analysis:** The electric field of a plane electromagnetic wave propagating in vacuum is given by:
    $\vec{E}(z, t) = (150 \text{ V/m}) \cos(1.0 \times 10^{15}\, t - 3.33 \times 10^{6}\, z)\, \overrightarrow{u_y}$

- What is the direction of propagation?
- What is the direction of polarization?
- Determine the angular frequency (ω), the wave number (k), the frequency (f), and the wavelength (λ). Verify that $\omega/k = c$.
- Write down the expression for the corresponding magnetic field $\vec{B}$(z, t).

11. **Spherical vs. Plane Waves:** Under what condition can a segment of a spherical wavefront be reasonably approximated as a plane wave? Explain why this approximation is often valid for light reaching Earth from the Sun.
12. **Intensity:** Using the wave from Exercise 10, calculate the time-averaged intensity (I) of the light wave in vacuum.
13. **Phase Difference:** For the wave in Exercise 10, consider two points on the z-axis: $z_1 = 0$ and $z_2 = 1.0\ \mu m$ (1.0×10^{-6} m). What is the phase difference $\Delta\varphi = \varphi(z_2, t) - \varphi(z_1, t)$ between the wave at these two points at any given instant t? Express your answer in radians.
14. **Coherence:** A certain type of laser has a coherence time (τ_0) of approximately 1.0 ns (1.0×10^{-9} s). Calculate its coherence length (l_0) in vacuum. Compare this to the typical coherence length of light from an incandescent bulb ($\tau_0 \approx 10^{-11}$ s). What does this difference imply for observing interference effects?
15. **light wave:** Adopting the approach followed by J.C. Maxwell , we can affirm that light is an electromagnetic wave characterized by the association of an electric field, a magnetic field and a wave vector . This statement obviously needs to be verified.

    In the vacuum the fields$\vec{E}$ and$\vec{B}$ which define the electromagnetic wave must satisfy the four equations of Maxwell (1860). Show that the combination of Maxwell's equations leads to the equations of propagation, electric and magnetic field
16. W**ave theory of light:** Either a linearly polarized wave. Of wave vector $\vec{k}_i$ falling on a flat and polished surface ( ) which separates two transparent media, of index $n_1$and n_2 :

    $$\vec{E}_i = \vec{E}_{0i} e^{i(\omega_i t - \vec{k}_i \vec{r})}$$

    If the reflected and transmitted waves exist and are associated respectively with the following fields:

    $$\vec{E}_r = \vec{E}_{0r} e^{i(\omega_r t - \vec{k}_r \vec{r})} \quad ; \vec{E}_t = \vec{E}_{0t} e^{i(\omega_t t - \vec{k}_t \vec{r})}$$

    To show that the conditions of continuity of the electric field make it possible to find the laws of reflection and refraction.

17. Either a flat electromagnetic wave placed in the vacuum in the field $\vec{E}$ is given by:

$$\begin{cases} \boldsymbol{E_x(z,t) = 10^2 cos\pi(9.10^{14}t - 3.10^6 z)} \\ \boldsymbol{E_y(z,t) = E_z(z,t) = 0} \end{cases}$$

- Give the amplitude of this wave, its direction of propagation and its polarization.
- Determine its speed, wavelength and frequency, what light is it
- Write the magnetic field expression $\vec{B}$ associated with this wave, make a propagation pattern indicating the wavefronts.

18. **Flat light wave:** The plane electromagnetic wave is considered to be: $\vec{E}(M,t) = \vec{E}_M cos\left[2\pi\left[5.10^{14}t - \frac{10^7}{12}(x+\sqrt{3}y)\right]\right]$

- Determine its period and frequency.
- Determine its direction of propagation in an Oxyz **direct orthonormed landmark.**
- Determine its wavelength and wave number.
- Calculate its propagation speed. Deduce the medium where the wave propagates.
- Calculate the components of the wave vector $\vec{\boldsymbol{k}}$

19. Light Wave from a Point Source: O source point emitting a monochromatic light wave in homogeneous, isotropic and transparent medium. The electric field at the source point written by : $E(O,t) = E_0 cos(\boldsymbol{\omega} t)$

- What is the shape of the wave surfaces?
- Give the expression of the wave surface whose phase difference with the source is ϕ.
- How to get a flat wave with a point source

Chapter II
Polarization of a light wave

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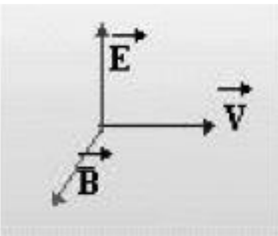
Introduction to Polarization of a Light Wave

The transverse nature of light, established via Maxwell's equations in Chapter I, distinguishes it fundamentally from longitudinal waves (like sound in air) and gives rise to the phenomenon of **polarization**. Polarization describes the orientation of the light wave's oscillations, specifically those of the electric field vector **E**, in the plane perpendicular to its direction of travel. This property is not just a theoretical detail; it is responsible for the function of LCD screens, polarized sunglasses, and various scientific instruments. In this chapter, we will systematically define the different polarization states – unpolarized, linear, circular, and elliptical – and develop the mathematical tools using field components and phase differences to describe them. We will then investigate practical methods for producing and modifying polarized light using polarizers (analyzing transmission with Malus's Law) and birefringent retardation plates (half-wave and quarter-wave plates).

II.1. Vector model of light

According to electromagnetic theory, luminous phenomena can be explained by the simultaneous propagation of an electric field **E** and a magnetic field **B**, which are constantly perpendicular to each other and to the direction of propagation, and whose values for a monochromatic wave are sinusoidal functions of time t.

At each instant, the vibration of the electric and magnetic fields takes place in a direction perpendicular to the direction of propagation of the light: this is called the wave plane. (P) this plane perpendicular to the "light beam".



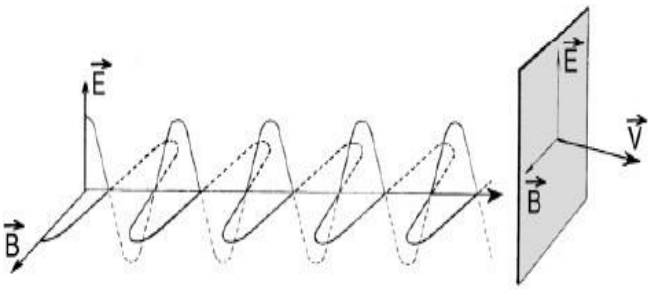


Figure II.1: propagation of an electric field **E** and a magnetic field **B**

II.2. Polarization states of light

In the following, we will only be interested in the electric field:

Thanks to Maxwell's Equations, if we know E, we know B

A wave is said to be unpolarized if $\vec{E}$ has a direction that varies randomly in the plane this is the case for **natural (unpolarized) light**. By convention, the polarized state of light is represented by a double arrow, representing the direction of oscillation of the electric field.For unpolarized light, this arrow has a random direction in the wave plane.

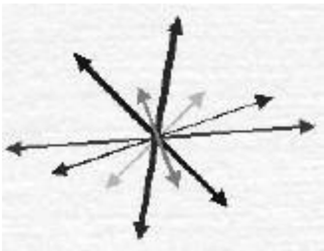


Figure II.2: Direction of electric field for unpolarized light

II.2.1. Polarized light

We know that $\vec{E}$ is contained in the wave plane. The extremity of this vector evolves in time according to a certain curve that defines the nature of the wave's polarization (state of polarization). A wave is said to be polarized if the components of the electric field vector have a

phase relationship.

Description: Consider a monochromatic travelling plane electromagnetic wave propagating inthe direction and sense of u_z to describe the field, we place ourselves in the xy plane and describe the evolution of the vector $\vec{E}$

$$E_z(z,t) = 0,\ \ E_y(z,t)\ E_{0y}\cos(\omega t - k.z + \varphi_1),$$

$$E_x(z,t) = E_{0x}\cos(\omega t - k.z + \varphi_2)$$

Simplifying by choosing the time origin appropriately leads to the form used predominantly:

$E_x(z,t) = E_{0x}\cos(\omega t - k.z)$

$E_y(z,t) = E_{0y}\cos(\omega t - k.z + \varphi)$

Where $\boldsymbol{\varphi} = \boldsymbol{\varphi_2} - \boldsymbol{\varphi_1}$ is the crucial **relative phase difference** between the y-component and the x-component. Depending on the phase shift and the **and Amplitudes** E_{0x}, E_{0y}, we can have different polarizations state:

❖ If $\boldsymbol{\varphi} = \boldsymbol{\varphi_2} - \boldsymbol{\varphi_1}$ is a multiple of π, the wave is said to be **rectilinearly polarized**: the **Linear Polarization** direction depends on arctan(E_{oy}/E_{ox}), $\vec{E}$ is constant.

For linearly polarized light, the end of the vector $\vec{E}$ describes a straight line segment in the wave plane. In space, the end of the vector describes a sinusoid

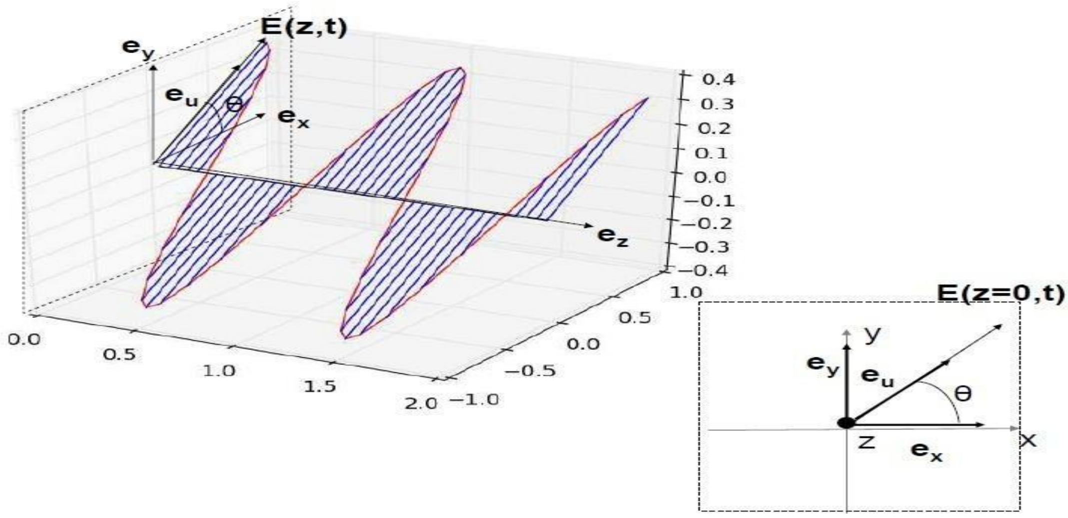
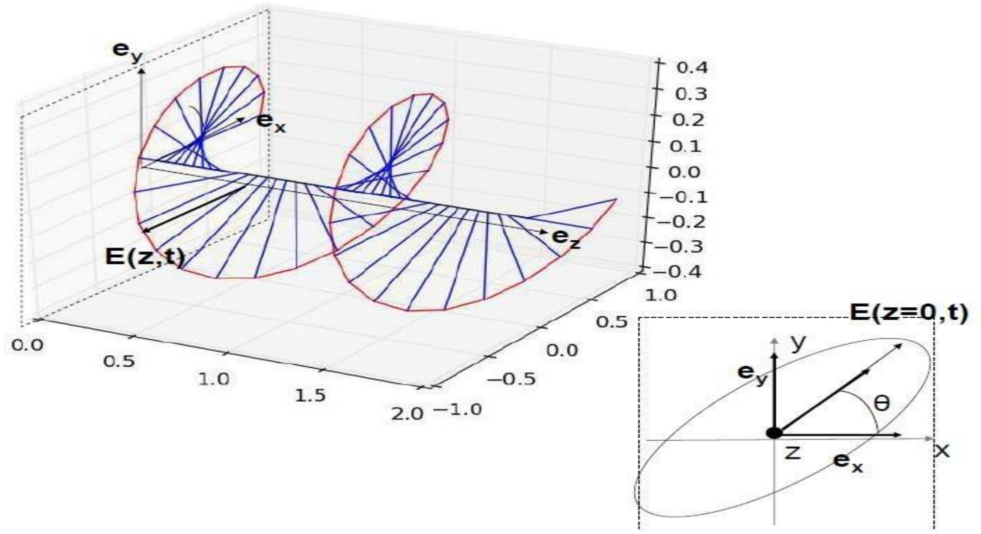


Figure II.3: Linear polarization

❖ If $\pi < \boldsymbol{\varphi} < 2\pi$, the end of the field describes an ellipse in the trigonometric direction: polarization is then said to be **right-elliptical** for an observer watching the wave advance towards him; if $0 < \boldsymbol{\varphi} < \pi$, the end of the field describes an ellipse in the clockwise direction. shows: polarization is said to be **left-elliptical**.

A wave is elliptically polarized if the end of its electric field vector $\vec{E}$ describes, over time, an ellipse in the wave plane P. The origin of the vector $\vec{E}$ is at the center of the ellipse. In space, the end of vector **E** describes an elliptical helix pitch.



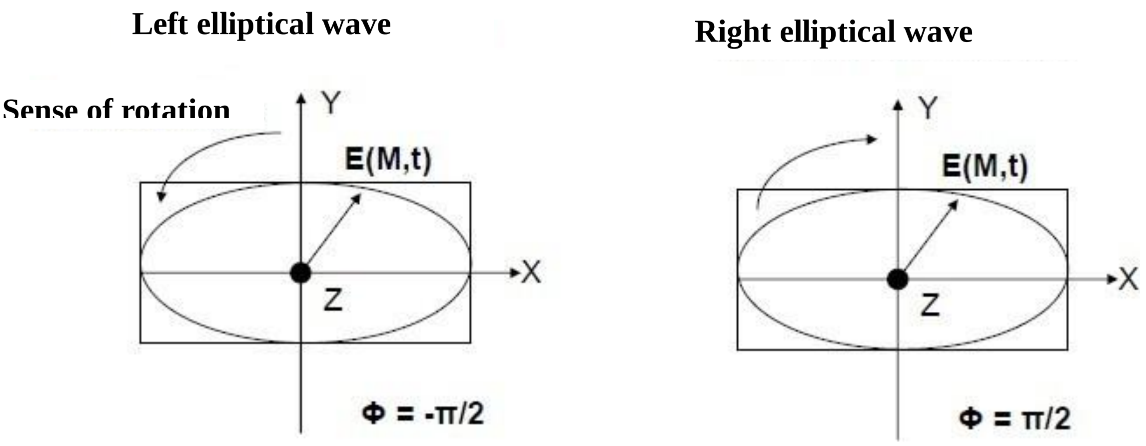


Figure II.4: Elliptical polarization of light

❖ Polarization is said to be **left circular** when $\varphi = \pi/2$ (the end of the field describe circle) in the trigonometric direction) and it is said to be **right circular** when $\varphi = 3\,\pi/2$ (The end of the field describes a clockwise circle. A wave is circularly polarized if the end of its electric field vector $\vec{E}$ describes, over time, a circle in the wave plane P. The origin of the vector $\vec{E}$ at the center of the circle. In space, the end of the vector $\vec{E}$ describes a circular helix pitch.

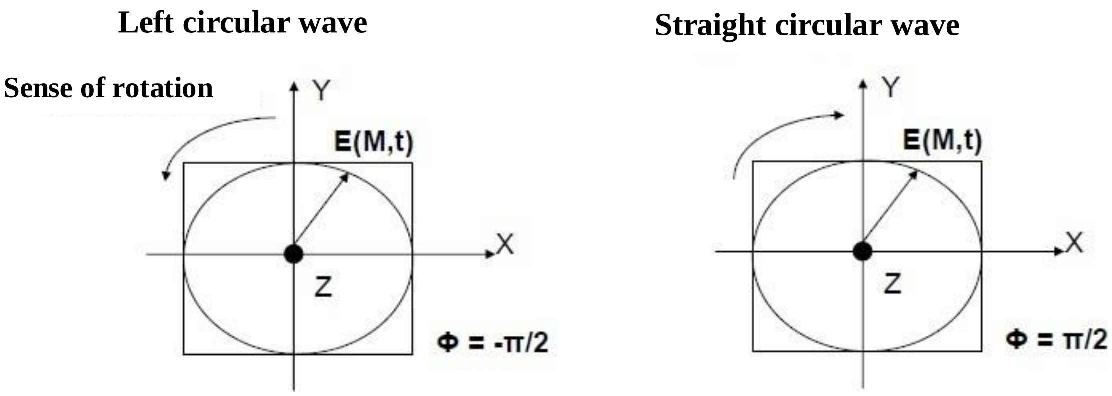


Figure II.5: Circular polarization of light

II.2.2. General case: in the general case, the wave is elliptically polarized

By eliminating time from both equations (1) and (2), we obtain the equation for the curve we're looking for:

$$\left(\frac{E_x}{E_{0x}}\right)^2 + \left(\frac{E_y}{E_{0y}}\right)^2 - 2\left(\frac{E_x}{E_{0x}}\right)\left(\frac{E_y}{E_{0y}}\right)\cos\varphi = \sin^2\varphi$$

It mathematically demonstrates how the amplitudes and phase difference define the shape and orientation of the polarization ellipse (or line/circle as special cases).:this is a common situation, similar to the Lissajous curves obtained in electronics.

II.3. Polarized light generation

For low-frequency EM waves (radio, microwaves...) with frequencies below 10 GHz => rectilinear, dipole antennas..

For "high-frequency" waves, and in particular optical waves, there are practically no natural"sources that are totally linearly polarized.

Solution: we'll "filter" the polarization directions.

II.3.1. Polarizer and Analyzer

Definitions:

Polarizer is an instrument that transforms an "unpolarized" electromagnetic wave, or one of any elliptical polarization, into a wave that is linearly polarized, parallel to one axis of the polarizer, called the transmission axis, while the orthogonal axis, located in the plane of the polarizer, is called the extinction axis.

- Polarizer: characterized by a polarization direction e_p, allows the incident field component parallel to e_p to pass through and eliminates the component perpendicular to e_p.

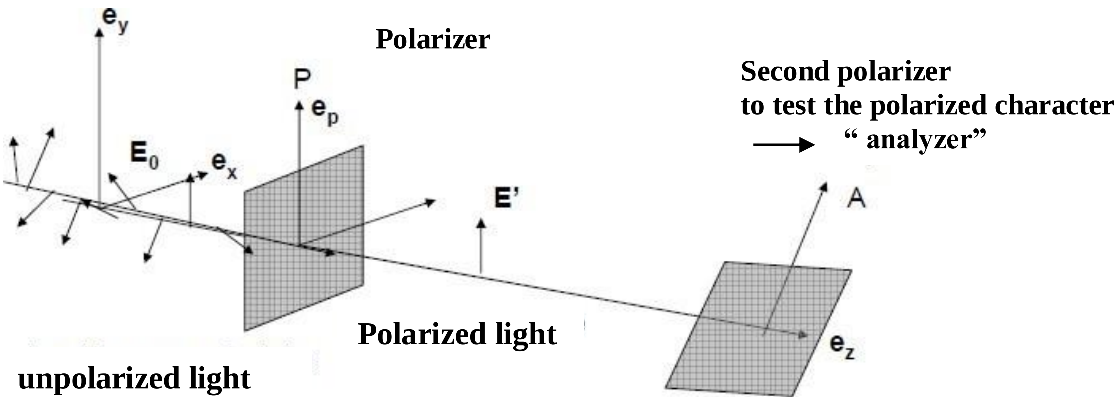


Figure II.6: polarization of light by polarizer and analyzer

- An analyzer is used to determine the direction of polarization of a linearly polarized wave; by changing the direction of the analyzer, extinction is observed when the **direction of the analyzer** is perpendicular to the direction of polarization of the wave.

a. Rectilinear polarization and Malus's law

Malus' law, named after Étienne Louis Malus, relates to the amount of light intensity transmitted by a perfect polarizer.
Suppose a plane wave linearly polarized by a first polarizer passes through a second polarizer (or analyzer). Note θ, the angle this polarization makes with the axis of the second polarizer.

Applies ONLY when the light incident on the analyzer is *already linearly polarized*. I_0 is the intensity of this incident linearly polarized light, and θ is the angle between its polarization direction and the analyzer's transmission axis. I is the transmitted intensity.

- *Crucial Note:* If starting with *unpolarized* light of intensity I_unpol, it passes through a first polarizer. The transmitted intensity *after this first polarizer* will be $I_1 = \frac{1}{2}$ I_unpol (linearly polarized). This I_1 then becomes the I_0 for Malus's Law when the light encounters a second polarizer (the analyzer) oriented at angle θ relative to the first: $I_2 = I_1 \cos^2\theta = (\frac{1}{2}$ I_unpol$) \cos^2\theta$.

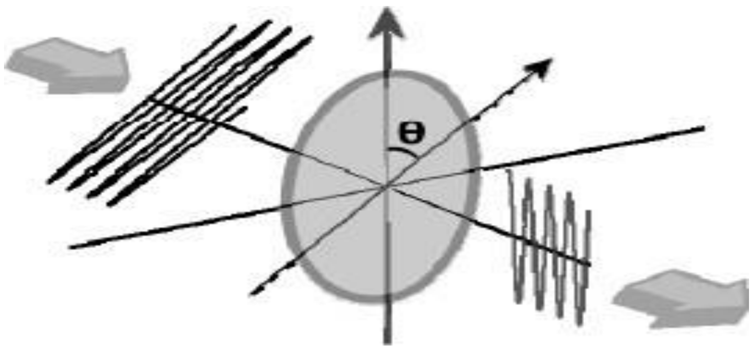


Figure II.7: Linear plane wave polarized by analyzer

The outgoing wave is then polarized along the axis of the second polarizer, but attenuated by acertain factor :

If we denote I_0 and I the incident and outgoing intensities of the analyzer, then Malus's law Written: $I = I_0 \cos^2 \theta$

This law has some important consequences:

- If the polarization of t h e incident wave is in the same direction as the analyzer axis, then all the light intensity is transmitted (θ = 0, so $I = I_0$).
- If the polarization of the incident wave is orthogonal to the polarizer axis, then there isno outgoing wave (θ = 90°).
- If the incident wave is unpolarized, i.e. made up of all possible polarizations, then averaging I gives $I = I_0/2$: half the intensity passes through. This is what we see when we look at a light bulb through a polarizer.

II.3.2. Retardation Plates

Definition: these are thin plates cut from a crystal with anisotropic optical properties, which modify the polarization state of an electromagnetic wave. Theorem: a retardation plate of thickness **e** and whose indices of the slow and fast axes are $\mathbf{n_0}$ and $\mathbf{n_e}$ respectively introduces an advance of E_x on E_y (or a retardation of E_y on E_x) of:

$$\varphi = \frac{2\pi}{\lambda_0}\delta \quad with\ \delta = (n_0 - n_e)e \quad (we\ assume\ n_0 > n_e)0$$

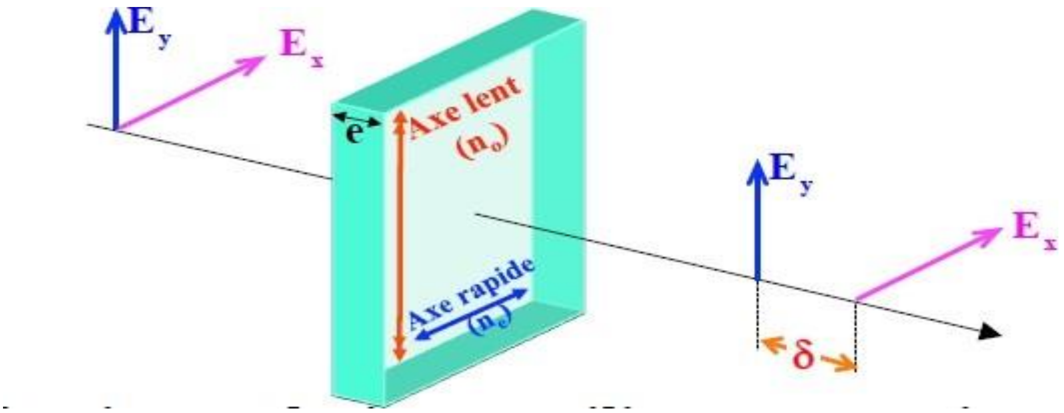


Figure II.8: retardation plate

- we call a half-wave plate for an electromagnetic wave of wavelength in vacuum λ_0 , a delay blade such that: $|\varphi| = \pi \Rightarrow |\delta| = \frac{\lambda_0}{2}$

II.3.2.1 Properties of a half-wave plate

1. a rectilinearly polarized incident wave is transformed into a rectilinearly polarized wave such that the direction of its electric field E_s is symmetrical to that of the electric field E_i ofthe incident wave with respect to the blade axes
2. a left-hand circularly polarized incident wave is transformed into a right-hand circularly polarized wave, and vice versa
3. an incident wave circularly polarized to the left is transformed into a wave circularly polarized to the right, and vice versa.

- we call a quarter-wave plate for an electromagnetic wave of wavelength in vacuum λ_0 a delay plate such that: $|\varphi| = \frac{\pi}{2}$ that is to say: $|\delta| = \frac{\lambda_0}{4}$

II.3.2.2. Properties of a quarter-wave plate

A circularly polarized incident wave is transformed into a linearly polarized wave along one ofthe bisectors of the blade axes, and vice versa.

$$|\varphi| = \frac{\pi}{2} \Rightarrow |\delta| = \frac{\lambda_0}{4}$$

II.3.3. Other Polarization Methods

II.3.3.1 Polarization by Reflection (Brewster's Angle):

When unpolarized light hits a boundary between two dielectric media (like air and glass), the reflected light is generally partially polarized. At a specific angle of incidence, called **Brewster's angle (θ_B)**, defined by $\tan(\theta_B) = n_2/n_1$ (where n_1 is the index of the incident medium and n_2 is the index of the refracting medium), the reflected light is **100% linearly polarized** with its E field oscillating parallel to the reflecting surface. At this angle, the reflected ray and the refracted ray are perpendicular (90° apart). This phenomenon is used in Brewster windows (e.g., on lasers) to transmit one polarization perfectly with no reflection loss.

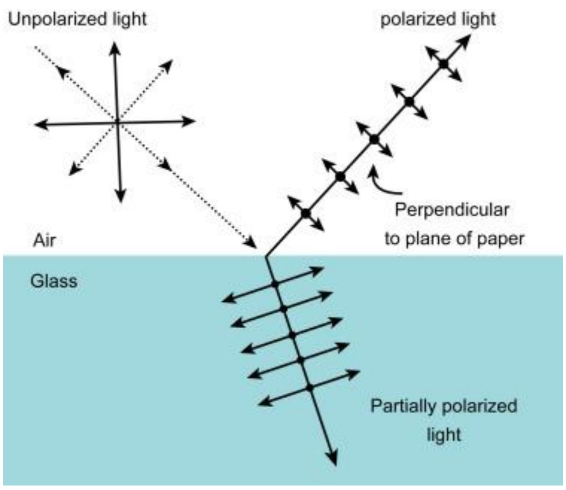


Figure II.9: the Browster's law polarization

II.3.3.2. Polarization by Scattering:

When light (especially shorter wavelengths like blue) strikes small particles (like air molecules), it gets re-radiated in various directions (scattered). The scattered light tends to be polarized. If unpolarized sunlight enters the atmosphere, the light scattered at 90° to the sun's direction is strongly linearly polarized. Looking at the blue sky through a polarizer and rotating it demonstrates this effect. This is why polarizing filters can darken a blue sky in photography, enhancing contrast with clouds.

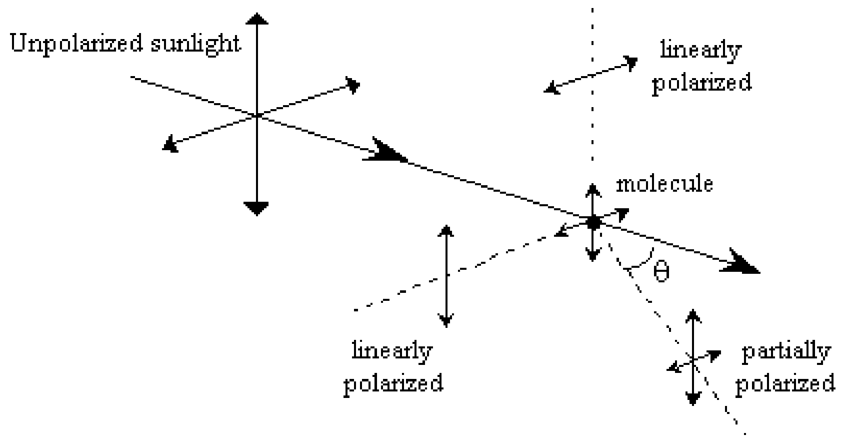


Figure II.10: Polarization by scattering

II.4. Conceptual Questions and Exercises for Chapter II

1. **Definitions:** Explain the difference between unpolarized, linearly polarized, and circularly polarized light in terms of the behavior of the electric field vector.
2. **Malus's Law I:** Unpolarized light with intensity I_unpol passes through three linear polarizers. The transmission axis of the second polarizer is at 30° to the first, and the axis of the third polarizer is at 90° to the *first* (i.e., 60° to the second). What is the intensity of the light emerging from the third polarizer in terms of I_unpol?
3. **Malus's Law II:** Linearly polarized light with an intensity of 100 W/m² is incident on a linear polarizer. The transmission axis of the polarizer makes an angle of 60° with the direction of polarization of the incident light. What is the intensity of the light transmitted through the polarizer?

4. **Quarter-Wave Plate:** Linearly polarized light is incident on a quarter-wave plate. Describe the output polarization state if the incident polarization is:
   (a) Parallel to the fast axis of the plate.
   (b) At 45° to the fast axis of the plate.
   (c) At 30° to the fast axis of the plate.
5. **Half-Wave Plate:** Linearly polarized light is incident on a half-wave plate with its polarization direction making an angle of 20° with the fast axis. What is the polarization state and orientation of the emerging light?
6. **Brewster's Angle:** Calculate Brewster's angle for light incident from air (n_1=1.00) onto water (n_2=1.33). What is the polarization state of the light *reflected* at this angle?
7. **Identifying Polarization:** You are given a light source and a linear polarizer. How could you determine if the source emits: (a) unpolarized light, (b) linearly polarized light, or (c) circularly polarized light? (Hint: How does the transmitted intensity change as you rotate the polarizer?)
8. **Conceptual:** Why can longitudinal waves (like sound in air) not be polarized?
9. **Quarter-Wave Plate I (Linear to Circular):** A quarter-wave plate ($\lambda/4$ plate) is designed for light of wavelength 500 nm. Linearly polarized light at 500 nm is incident normally on the plate. The polarization direction of the incident light makes an angle of 45° with the fast axis of the plate. Describe the polarization state of the emerging light. Be specific about the type and, if applicable, the handedness (assuming the y-component lags the x-component if the phase shift $\varphi = -\pi/2$, corresponding to right-handedness).
10. **Quarter-Wave Plate II (Other Input):** What is the polarization state of the light emerging from the quarter-wave plate in Question 5 if the incident linearly polarized light makes an angle of:
    (a) 0° with the fast axis?
    (b) 90° with the fast axis?
    (c) 30° with the fast axis?
11. **Half-Wave Plate (Rotation):** Linearly polarized light is incident normally on a half-wave plate ($\lambda/2$ plate). The incident polarization direction is at 35° relative to the plate's fast axis. What is the polarization state and orientation of the emerging light relative to the fast axis?

12. **Half-Wave Plate (Circular Input):** Right-circularly polarized (RHC) light is incident normally on a half-wave plate. What is the polarization state and handedness of the emerging light?
13. **Brewster's Angle:**

    (a) Calculate Brewster's angle for light incident from air ($n_1 \approx 1.00$) onto dense flint glass ($n_2 \approx 1.66$).

    (b) Describe the polarization state of the light that is *reflected* when unpolarized light is incident at this angle.
14. **Identifying Polarization States I:** You have a device that emits a beam of light. You also have a single sheet of linear polarizing material. How can you use *only* the polarizing sheet to determine whether the light emitted by the device is:

    (a) Unpolarized?

    (b) Linearly polarized?

    (c) Circularly polarized?

    (Explain what you would do and what intensity variations you would observe in each case as you rotate the polarizer).
15. **Identifying Polarization States II**: A light beam of pulse ω and wavelength λ in the following Oz propagating vacuum is described by:

    $$E_x = E_0 \cos(\omega t - kz), E_y = E_0 \sin(\omega t - kz), E_z = 0 \quad with\ k = \omega/c = 2\pi/\lambda$$

    (a) Show that it is a circular vibration whose direction of travel will be specified.

    (b) An analyzer is placed in the plane in the plane z = 0; its polarization direction makes an angle θ with Ox. Determine the E field at the output of the analyzer. How does the light intensity I after the analyzer vary according to θ?

    © Repeat the previous questions for the incident field: $E_x = E_0 \cos(\omega t - kz), E_y = -E_0 \sin(\omega t - kz), E_z = 0$
16. **Retardation Plates**: Either a single-axis blade, of thickness e, cut parallel to its optical axis is illuminated by a parallel and monochromatic beam of wavelength λ, in normal incidence.

    (a) Calculate the phase differenceφ introduced between ordinary and extraordinary vibrations after passing through the blade.

    (b) What thickness must the blade take for it to be a quarter waves for the given radiation?

(c) The electric field $\vec{E}$ of the incident wave makes an angle α with the optical axis of the blade. What is the nature of emerging vibration? discuss special cases. $\lambda=0.589\mu m$, $n_0-n_e=0.004$

Chapter III

Light interference (The interference to two waves)

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Introduction to Light Interference

This chapter investigates **light interference**, a phenomenon fundamentally demonstrating the wave nature of light through the principle of superposition. When coherent light waves meet, their superposition results in a spatially varying intensity pattern, with regions of constructive (bright) and destructive (dark) interference. We will mathematically analyze the superposition of two waves to determine the resultant intensity, highlighting the importance of the phase difference. Key conditions for interference, particularly coherence, will be discussed, along with the major methods of producing interfering waves: wavefront division (e.g., Young's slits) and amplitude division (e.g., Michelson interferometer, thin films). Concepts such as optical path difference, fringe order, contrast, and the role of coherence length will also be explored.

III.1. Definition

Interference occurs when the superposition of two or more waves leads to a light intensity that is different from the sum of the intensities of each of the waves taken separately.

Conditions for observing interference:

- The waves must have the same frequency
- Polarizations of interfering waves should not be orthogonal
- The observation of interference is related to the phase difference of the waves: This difference phase must be constant during the observation time.

III.2. Overlay of two waves

We are interested in the field observed by a detector located at point M when two waves «1» and «2» arrive in M.

superposition of two or more waves. When waves overlap at a point M, the total instantaneous field S(M, t) is the sum of the individual fields $S_1(M, t)$ and $S_2(M, t)$. This is the Principle of Superposition.

Let be two sinusoidal waves of pulsations respective ω_1 et ω_2, which propagate in the same direction. An M point receives both waves:

$$S_1(M,t) = A_1 \cos(\omega_1 t - \varphi_1(M))$$
$$S_2(M,t) = A_2 \cos(\omega_2 t - \varphi_2(M))$$

The field in M is the superposition of the fields of the two waves 1 and 2

$$S(M,t) = S_1(M,t) + S_2(M,t)$$

The light intensity ***I(M)*** transported by a light wave will be proportional to the time average of the square of the total field, then the module is calculated to the square of the total field S:

$$I(M) = \langle |S(M,t)|^2 \rangle = \langle |S_1(M,t) + S_2(M,t)|^2 \rangle$$

$$I(M) = \langle |S_1(M,t)|^2 + |S_2(M,t)|^2 + 2|S_1(M,t)S_2(M,t)| \rangle$$

$$I(M) = I_1 + I_2 + I_{1-2}$$

$$I_1 = \langle |S_1(M,t)|^2 \rangle, \;\; I_2 = \langle |S_2(M,t)|^2 \rangle, \quad I_{1-2} = \langle 2|S_1(M,t)S_2(M,t)| \rangle$$

$$I_{1-2} = \langle 2|S_1(M,t)S_2(M,t)| \rangle = 2A_1A_2 \langle \cos(\,\omega_1 t - \varphi_1(M)) \cos(\,\omega_2 t - \varphi_2(M)) \rangle$$

Using trigonometric identities and time-averaging, this yields the interference term I_{1-2}:

$$I_{1-2} = 2A_1A_2 \langle \cos[(\omega_1 + \omega_2)t - (\varphi_1(M) + \varphi_2(M))] \rangle \\ + 2A_1A_2 \langle \cos[(\omega_1 - \omega_2)t - (\varphi_1(M) - \varphi_2(M))] \rangle$$

The average value of $\cos(wt - \emptyset)$ is not null except for $w = 0$ when $w = \omega_1 + \omega_2$ or $w = \omega_1 - \omega_2$.The first term is always null, the second term is not null if $\omega_1 = \omega_2$ so:

$$I(M) = I_1 + I_2 + 2\sqrt{I_1 I_1} \langle \cos(\varphi_2(M) - \varphi_1(M)) \rangle$$

The final formula I(M) is the cornerstone of two-beam interference. It clearly shows that the resulting intensity is *not* simply the sum $I(M) \neq I_1 + I_2$ but includes an additional term that depends on the phase difference between the waves and There can only be moderate interference between light waves if they are coherent $\omega_1 = \omega_2$.

- ❖ The intensity $I(M)$ depends on PHASE DIFFERENCE $\varphi_2(M) - \varphi_1(M)$ which itself depends on the position M.

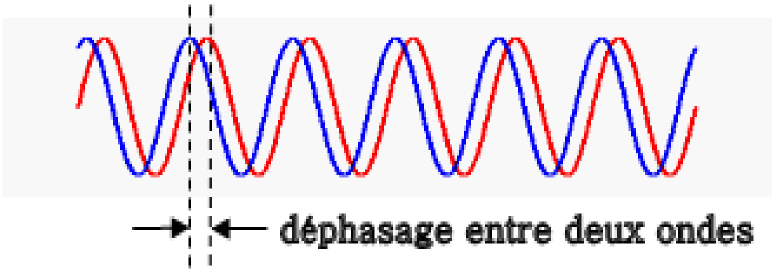


Figure III.1: Phase shift between

- $I(M)$ is maximum if $\varphi_2(M) - \varphi_1(M) = 2\pi p$. The points verifying this relationship form a BRILLIANT IMAGE.

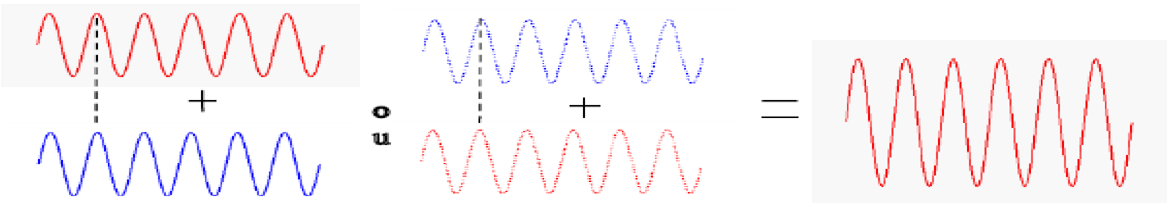


Figure III.2 : Constructive interference

- $I(M)$ is minimum if $\varphi_2(M) - \varphi_1(M) = 2\pi p + \pi$. The points verifying this relationship form a DARK FIELD.

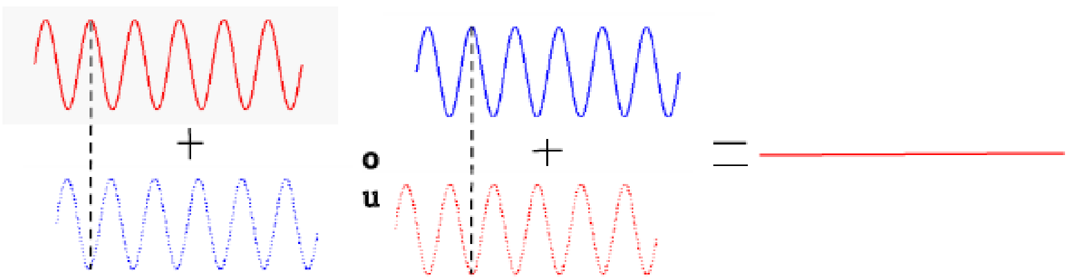


Figure III.3 : Destructive interference

Practically, to achieve interference, we start from a single source, from which we divide the emergent wave using a suitable optical device called *beam separator*.

There are many models, which it is convenient to divide into two large families: amplitude dividers and wave front dividers.

III.3. Wave front Division:

One method is to divide the wave packet spatially using an optical system in a region where it has a large spatial coherence width. Two-wave interference by dividing the wave front results from the superposition of two waves from two coherent sources. Most of the interferential devices refer to the study of Young holes, the table below recalls systems equivalent to Young holes.

- **Young's double slit experiment :**

Thomas Young (1773-1829) was an English doctor who discovered interference by passing light from a point source through two small nearby holes (Figure III.4). These holes behave as two coherent sources emitting divergent beams by diffraction. On a remote screen, Young then observes interference fringes in the area of beam overlap. It performs measurements that it interprets on the basis of a wave model, which allows it to evaluate the wavelength of light for different colors, such as red (0.70 **mm**) and purple (0.42 **mm).**

Principle : Light from a small source illuminates two narrow, closely spaced slits. Interference is observed on a distant screen. Acts as a classic example of wavefront division. The slits act as coherent secondary sources.

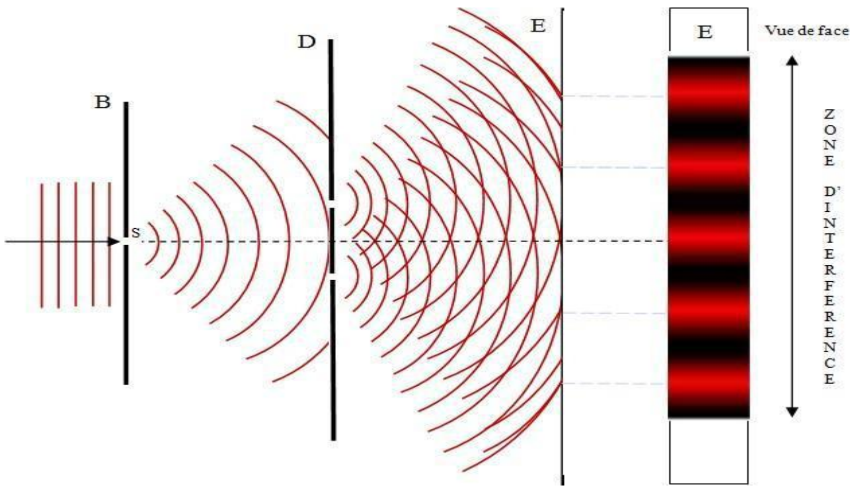
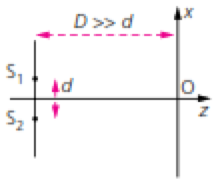
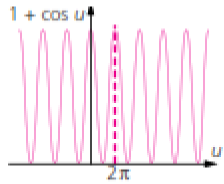


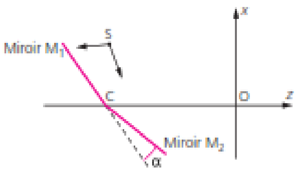
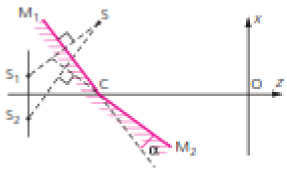
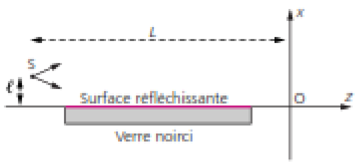
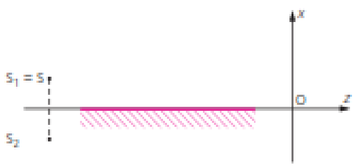
Figure III.4: Yong's interferential device

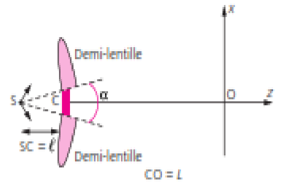
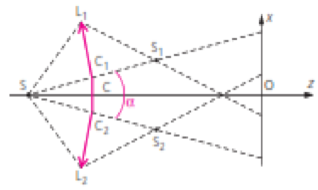
Results: Produces parallel bright and dark fringes on the screen. Fringe spacing $\beta = \lambda D/d$ (where λ=wavelength, D=slit-screen distance, d=slit separation).

III.3.1. Young's experiment study

Young's holes 2-wave interference		$I(x, y) = 2I_0\,(1 + \cos\gamma x)$ 	$\gamma = \frac{2\pi d}{\lambda D}$ note: if the screen (o,x,y) is placed in the focal point of a converging lens of focal length f', it suffices to replace D by f' in the expression of γ.

III.3.2. Systems equivalent to Young holes

types de pupilles	***Schéma du système***	***système équivalent***	***paramètres***
Fresnel mirrors			S1 and S_2 are the images of ***S*** respectively through M_1 and M_2. ***$d=S_1S_2=2CS.\sin\alpha$*** ***$D \approx CO$***
Lloyd's Mirrors			S_1 is confused with ***S***, and S_2 is the image of **S** through the looking glass. ***$d=2l$, $D=L$*** Sources S_1 and S_2 are phase shifted by ***p*** since the wave which has undergone a reflection on the mirror (from S_2) was phase shifted from ***p***, while that which reaches the screen directly (from S_1) does not undergo any phase shift.

Billet Bilentilla			S_1 is the image of ***S*** through the half-lens L_1 (center C_1) and S_2 the image of ***S*** through the half-lens L_2 (center C_2). $d = S_1S2 \approx l^2\alpha/(l-f')$ (for α low; f' is the focal length of L_1 and L_2) $D \approx SO$

III.4. Amplitude Division

Another method consists of splitting wave packets from an S source uses a partially reflecting surface (beam splitter) to divide the *amplitude* of an incident wave into two separate beams (one reflected, one transmitted). These beams travel different paths and are recombined to interfere. Can work with extended sources because each point on the source interferes only with itself via the two paths. Examples: Michelson Interferometer, Fabry-Perot Interferometer, interference in thin films.

III.4.1. Michelson interferometer

Several interferential devices, called **interferometers**, produce interference patterns of the same nature as thin sections. They are therefore devices with amplitude division, which can use wide sources of light. The most important of them, both from a practical and historical point of view is **the Michelson interferometer**, named after its designer, Albert Michelson, 1st American Nobel Prize in Physics in 1907. The Michelson interferometer is an extremely precise device, allowing obtaining very important differences in gait. Very versatile, its applications range from stellar interferometer (developed by Michelson himself) to determine the spectral stellar interferometry composition of the light emitted by stars, the control of polishing of flat or parabolic mirrors or the very precise measurement of the thickness of a blade with parallel faces.

Principle: Amplitude division using a beam splitter. Light splits, travels to two mirrors (M_1, M_2), returns, recombines, and is observed. One mirror is typically movable. Often includes a compensator plate for white light use. Interference occurs between the beams reflected from M_1 and M_2 (or rather, the image of M_2 as seen through the beam splitter) Figure III.5. The **O**ptical **P**ath **D**ifference depends on the difference in arm lengths and the angle of observation.

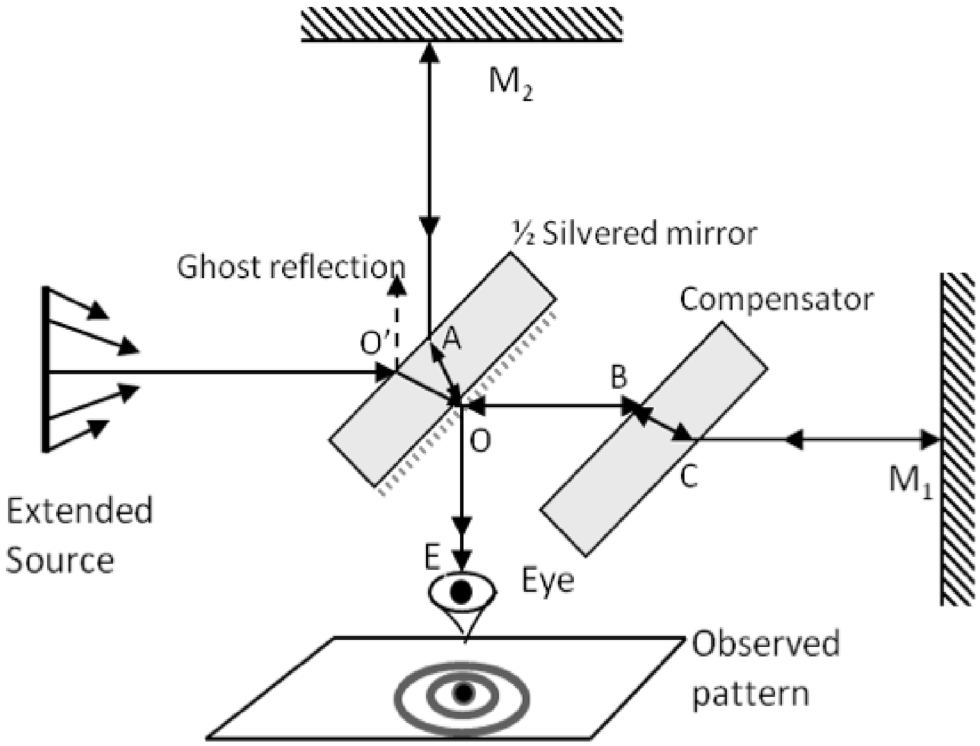
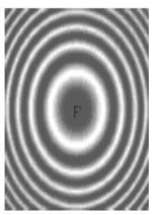
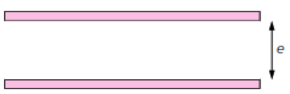
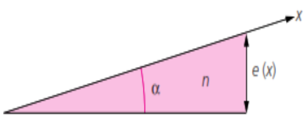
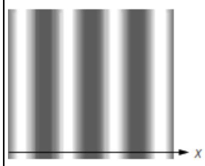
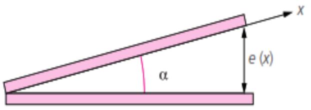
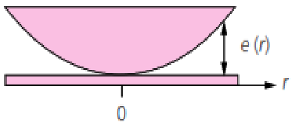
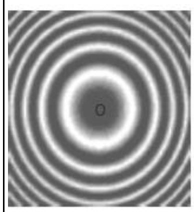


Figure III.5: Michelson interferometer

III.4.2. Interference by aplitude division

Blade type	Fringe type	Fringe shape	Location	Interference order
Parallel-sided glass slide 	*Fringes of equal inclination*	*Rings* 	*at infinity or in the focal plane image of a converging lens*	$p = \frac{cosr}{\lambda} + \frac{1}{2}$ *(if i designates the angle of incidence of the wave on the blade, r is the angle of refraction with sin i 5 n sin r)*
Air blade with parallel faces 				$p = \frac{cosi}{\lambda}$
Glass blade of variable 		*Straight bangs*	*in a plane close to the blade*	$p = \frac{2n\alpha}{\lambda} + \frac{1}{2}$

thickness	*Fringes of equal thickness*		*(exterior or interior to the blade) in a plane close to the blade*	
Air wedge with two flat faces 				$p = \frac{2n}{\lambda} x$
Air wedge with one spherical face (radius R) and one flat face 	*Fringes of equal thickness*	*Newton's rings* 	*in a plane close to the blade*	$p = \frac{r^2}{R\lambda} + \frac{1}{2}$ *(r is the distance to O on the screen)*

III.5. Difference in performance

The **difference in step between** two light rays represents the difference in the **optical paths traversed by these two rays.**

For two coherent waves, the intensity at a point M is written:

$$I(M) = I_1 + I_2 + 2\sqrt{I_1 I_1}\langle\cos(\varphi_2(M) - \varphi_1(M))\rangle$$

THE PHASE DIFFERENCE $\quad \varphi_2(M) - \varphi_1(M) = \varphi_{02} - \varphi_{01} + \frac{2\pi}{\lambda}(L_{s2} - L_{s1})$

L_{s2} and L_{s1} are the optical paths traversed by two light rays that overlap in M.

The difference of Walk: $\delta_M = (L_{s2} - L_{s1})$ depends on the point M: one expects to observe a non-uniform illumination.

III.6. Interference fringe: surface of equal intensity

Experimentally, to characterize the interferences, we define the interference fringes as the surfaces where the intensity $I(M)$ is constant (are geometric curves on a plane of observation).

III.7. The interface

The interfrange i is the distance of two consecutive fringes of the same nature

a. Position of shiny fringes

A point $M(x)$ belongs to a bright fringe if the difference of walk between the two waves that arrive in M is= with k

b. Position of dark or dark fringes

A point M belongs to a dark fringe if the difference in walk δ in between waves that arrive in M is: $\delta = (2k+1)\frac{\lambda}{2}$ with $k \in Z$

III.8. interference order

The order of interference at point M is defined as: $P(M) = \frac{\varphi_2(M)-\varphi_1(M)}{2\pi}$

It varies by one unit each time the phase difference between the vibrations varies from 2π surface of equal intensity:the resulting intensity $I(M)$ ***depends only on the phase difference*** $\varphi_2(M) - \varphi_1(M)$ so the surfaces of equal intensity can be defined by $P(M) = Cste$

$$I_{Max} \rightarrow P \text{ is an integer}$$

$$I_{Min} \rightarrow P \text{ is a half integ}$$

III.9. Fringe contrast (Visibility)

The contrast between a maximum and a minimum intensity is evaluated by the relationship..:

$$V = \frac{I_{Max} + I_{Min}}{I_{Max} - I_{Min}}$$

$$I_{Max} = I_1 + I_2 + 2\sqrt{I_1 I_1}$$

$$I_{Min} = I_1 + I_2 - 2\sqrt{I_1 I_1}$$

V is maximum when the interfering waves are coherent and have the same amplitude. It is null when the waves are not coherent.

III.10. Time coherence

The light emitted by a luminous point can be described as a succession of identical τ duration $\tau(10^{-6}\ to\ 10^{-10}\ s)$ wave-selective trains, emitted at random moments by the source point.

The emitted wave trains, having a finite length $l_c = c\tau$, give interferences only if the wave train from a secondary source is superimposed on that from the other secondary source.

If the wave trains cannot overlap in the observed area, light interference cannot be created

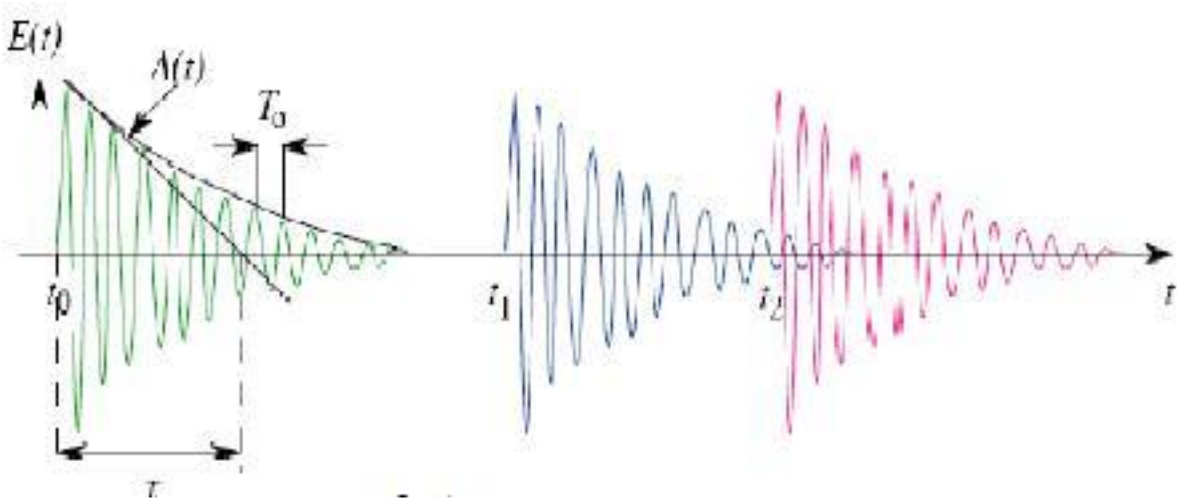


Figure III.6: Wave train emitted by the source

A necessary condition for the observation of interference at a point is that the difference of the two optical paths traversed between the point source and the point is less than the coherence length of the source

The condition of temporal coherence results in: $\delta \leq l_c$

With δ the optical path difference between the two waves at the point of observation. If the wave trains cannot overlap in the observed area, light interference cannot be created.

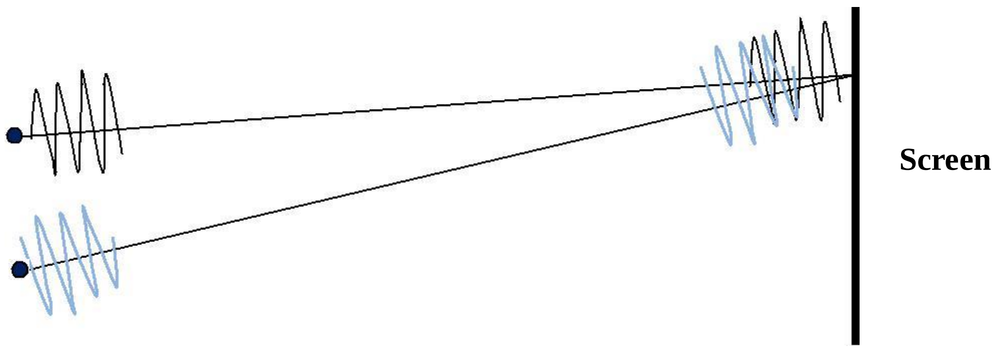


Figure III.7: Superposition of wave trains

In conclusion the conditions Required for Interference:

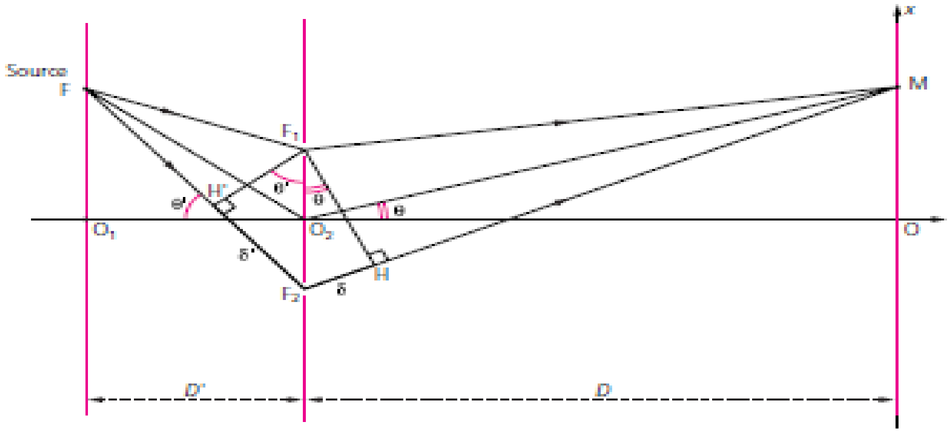
- Same Frequency ($\omega_1 = \omega_2$): Essential for the phase difference Δφ to be constant in time. If frequencies differ, the interference term $\cos((\omega_1\text{-}\omega_2)t\text{-}\ (\varphi_1\text{-}\varphi_2))$ would oscillate rapidly, averaging to zero over typical detection times, resulting in $I = I_1 + I_2$ (no visible fringes). This is linked to monochromaticity.
- Non-Orthogonal Polarizations: Interfering electric fields must have components vibrating in the same direction. If E_1 is purely x-polarized and E_2 is purely y-polarized, they superpose vectorially, but the intensity is just $I = I_1 + I_2$. Maximum visibility occurs when polarizations are identical.
- Coherence (Constant Phase Difference): This is the most critical condition. The phase difference Δφ between the waves arriving at point M must remain constant during the observation time.
- Temporal Coherence: Explained using the wave train model. Real sources emit finite wave trains of length $l = c\tau_0$. For interference to occur, the wave trains taking different paths must overlap temporally when they reach the detector. This requires the Optical Path Difference (OPD or δ) between the paths to be less than the coherence length: OPD ≤ l. Lasers have long l, making interference easy; thermal sources have short l, making it harder.
- Spatial Coherence: While not explicitly named, it's implied by the need to start from a single point/small source in wavefront division (like **Young's Double Slit Experiment**). This ensures different points across the initial wavefront have a fixed phase relationship, so the secondary sources created by division (e.g., the two slits) are coherent with each other.

III.11. Conceptual Questions and Exercises for Chapter III

1. **Basic Intensity Calculation:** Two coherent beams of light interfere at a point P. Beam 1 has intensity $I_1 = I_0$ and Beam 2 has intensity $I_2 = 4I_0$. What is the resultant intensity I(P) if the phase difference between the beams at P is:
   (a) Δφ = 0 radians?
   (b) Δφ = π radians?
   (c) Δφ = π/2 radians?
2. **Fringe Contrast:** In an interference experiment, the maximum intensity observed is $I_{max} = 16$ units and the minimum intensity is $I_{min} = 4$ units.
   (a) Calculate the fringe visibility (contrast) V.

(b) What is the ratio of the intensities of the two interfering beams (I_1/I_2)? (Assume $I_1 \geq I_2$).

3. **Lateral Source Shift in Young's Hole Experiment I:** The Young experiment is carried out as follows: a plate P is pierced by two identical narrow slots F1 and F2, parallel to each other and distant from a. A slit very fine source F is illuminated by a monochromatic light of wavelength λ. All slots are contained in planes perpendicular to that of the figure. Note ***that*** the distance from F to the center O1 of the plane S. S is located at a distance D' of P. At the distance D ≫ of the plane of the slots, a screen is placed, also vertical.
   (a) Be ***two rays from the source and arriving on the screen at a distance*** x to O, after passing through F1 and F2 respectively. Determine the difference in gait and phase shift between these rays.
   (b) How is the image on the screen changed from the usual situation where F is equidistant from both F1 and F2 slots? The position of the center of the interference figure will be given.



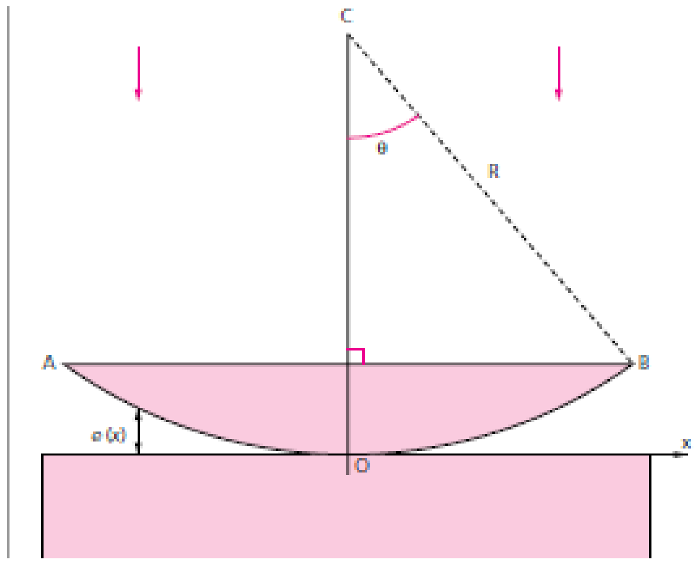
4. **YDSE II- Fringe Spacing and Position:** Using the setup from Question 3 (d=0.25 mm, D=1.20 m) but now using light with a wavelength λ = 500 nm:
   (a) Calculate the fringe spacing (interfringe distance) β.
   (b) Find the position y of the third dark fringe (the dark fringe corresponding to m=2 in the formula OPD = (m+1/2)λ).
5. **YDSE II- Effect of Medium:** If the entire YDSE setup described in Question 4 (d=0.25 mm, D=1.20 m, λ_0=500 nm in vacuum) is immersed in water (refractive index n = 1.33), what will be the new fringe spacing β_water observed on the screen?
6. **Newton's Rings:** We consider the device of Newton's rings. For this purpose a convex plane lens of radius R and angle of opening u is used. The lens rests by its curved face in O on a

plane of glass Ox. There is therefore between the plane and the lens a blade of air of variable e(x) thickness, with e(0) = 0. It is assumed that e remains weak in front of the radius R of the curved face. An extensive monochromatic source illuminates the lens in normal incidence.

(a) Recall which interfering waves and specify the location of the interference fringes.

(b) Calculate the d(x) step difference between two interfering rays.

(c) Express e(x) based on x; describe the interference pattern. What is the central fringe?



7. **Michelson Interferometer - Wavelength Measurement:** In a Michelson interferometer using a laser, one mirror is moved by a distance $\Delta e = 0.150$ mm. During this movement, 475 bright fringes are observed to pass the center of the field of view. Calculate the wavelength (λ) of the laser light.

8. **OPD and Phase Difference:** Two light rays travel different paths from a source to an observation point P. The optical path difference (OPD) between them is calculated to be 1250 nm. If the light has a vacuum wavelength $\lambda_0 = 500\ nm$:

   (a) Calculate the phase difference $\Delta\varphi$ between the rays at point P in radians.

   (b) What is the order of interference P at point P?

   (c) Will the interference at P be constructive, destructive, or intermediate?

9. **Coherence Length:** An interference experiment is performed using a light source with a coherence length $l = 1.5\ mm$. What is the maximum optical path difference (OPD) between the interfering beams for which interference fringes will still be clearly observable?

10. **Coherence Requirement:** A Michelson interferometer is set up such that the difference in its arm lengths is 5.0 cm. To observe interference fringes, what is the minimum required

coherence length (l) of the light source used? What kind of source (e.g., LED, laser, incandescent bulb) would likely be needed?

11. **Conceptual - Wavefront vs. Amplitude Division:** Explain the fundamental difference in how interfering beams are created in Young's Double Slit experiment versus a Michelson Interferometer. Why can the Michelson often use an extended light source while YDSE requires a point-like or line source?

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Chapter IV
Diffraction of a Light Waves

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Introduction to Diffraction of a Light Wave

This chapter addresses **diffraction**, the process by which a wave bends or spreads out as it passes by the edge of an object or through an aperture. As a fundamental property of wave propagation, diffraction limits the resolution of optical instruments and plays a key role in various optical technologies. We will utilize the Huygens-Fresnel principle—viewing points on a wavefront as sources of interfering secondary wavelets—to understand how diffraction patterns arise. The distinction between near-field (Fresnel) and far-field (Fraunhofer) diffraction regimes will be discussed, followed by an analysis of the characteristic Fraunhofer diffraction patterns from slits and rectangular apertures.

IV.1. Definition

Diffraction is any deviation of the light rays from their trajectory that cannot be explained by a phenomenon of reflection or refraction (**Limits of Geometric Optics)**. The diffraction phenomenon can also be observed when the path of light is altered by a screen edge. The study of diffraction, in the general case, is complex, a number of approximations are then necessary.

The diffraction phenomenon becomes observable only if the obstacle size is smaller at the wavelength of the incident light.

IV.2. Huygens-Fresnel principle

The Huygens-Fresnel principle lays the foundations for the interpretation of diffraction phenomena. It is a good illustration of the path of scientific thought since it uses ideas proposed more than a century apart by Huygens and Fresnel. It allows to interpret in a relatively simple way the diffraction.

- ❖ Contribution by Huygens (1678):

The light propagates from close to close. Each surface element reached by light behaves as a secondary source that emits spherical wavelets whose amplitude is proportional to the surface of the secondary source.

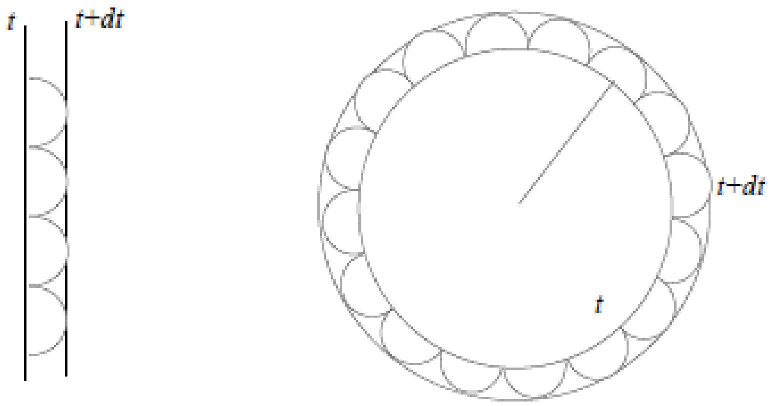


Figure IV.1: Evolution of wave surfaces in an isotropic medium for plane and spherical waves.

The wave surface at the time ***t+dt*** is deduced from the wave surface at ***t*** by considering at the time ***t+dt*** the envelope of the Huygens wavelets that propagated during the interval of time ***dt*** (Figure IV.1)**.**

This representation of the propagation of a wave is intuitively related to the propagation of waves on the surface of a liquid. This type of construction allowed Huygens to explain simply the phenomenon of light refraction. It is easy to imagine using the Huygens construction that a wave surface cannot propagate without deformation if it encounters an obstacle as shown in the figure below (Figure IV.2).

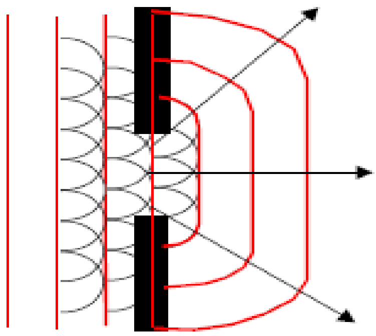


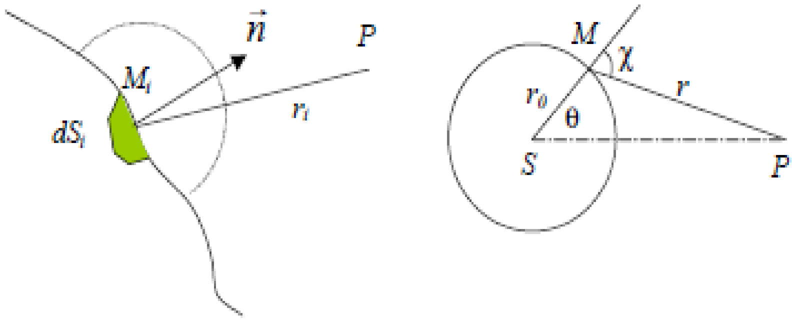
Figure IV.2: the Huygens construction

- ❖ Contribution of Fresnel (1818):

The complex amplitude of the light vibration at a point is the sum of the complex amplitudes of the vibrations produced by all secondary sources. The light vibrations interfere to form the resulting light at the P point under consideration.

More precisely, Fresnel cuts the wave front into surface elements acting as secondary sources and emitting spherical harmonic waves (Figure IV.3).

The construction of Huygens was supplemented by Fresnel's hypothesis that there may be interference between the different wavelets. In other words, the observed diffraction pattern results from the interference of the waves emitted by all the secondary sources uniformly distributed on the diffracting diaphragm.



FigureIV.3: Representation used by Fresnel to explain how to calculate the field at the observation point **P.**

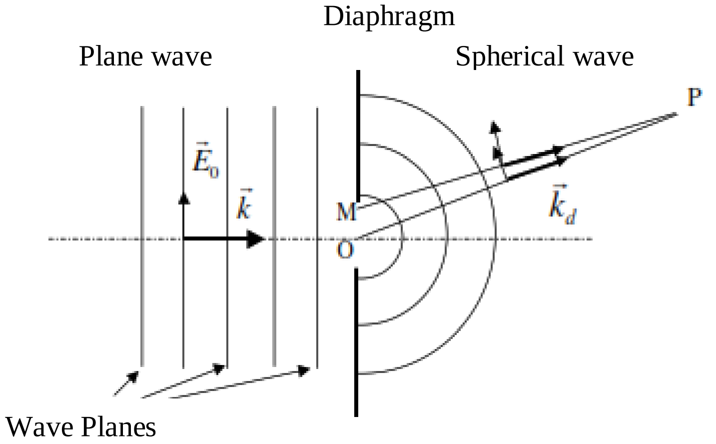
IV.3. Mathematical Formulation

Either an opaque screen pierced with an aperture of dimension ***d*** and illuminated by light vibration ψ coherent (Figure IV.4). If $\psi(M)$ refers to the complex amplitude of the wave produced in M and $\psi(P)$ the vibratory state of the secondary source located in P, and according to the Huygens-Fresnel principle the vibration in the observation plane (OXY) is written:

$$\psi(P) = \iint_S K\psi(M)\frac{e^{-ikr}}{r}dS$$

The term in $1/r$ means that we consider the propagation of spherical wavelets, the exponential term is a phase shift term between the source point M of the diffracting surface and the point P, the proportionality constant K est actually depends on the direction θ in which the wave is emitted.

It is generally placed in the case of the paraxial approximation, that is to say that we consider points P distributed on a small lateral extension before z. Moreover, Fresnel is placed in paraxial conditions so that the diffraction angles remain close enough to zero to be able to consider K as constant.



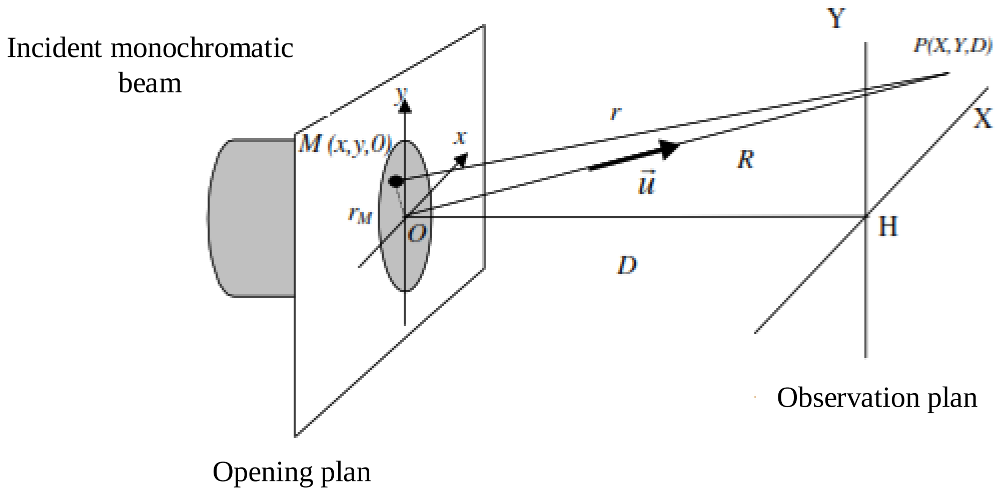


Figure IV.4: Transformation of a monochromatic plane wave into a spherical wave passing through a point O at aperture

- **Note:**
  - Field of study:

Either a monochromatic wave falling on a plane diaphragm. The wavelength is assumed to be much smaller than the dimensions of this aperture.

- **Finit distance diffraction or Fresnel diffraction:**

This is to describe the distribution of light intensity on a screen placed near the diffracting pupil.

- **Infinité diffraction or Fraunhofer diffraction:**

It is then a question of describing the distribution of light intensity on a screen placed at a great distance from the diffracting pupil, theoretically to «infinity».

- **Pupil**

When the objects under consideration are flat openings pierced in opaque screens, they are referred to **as pupils or diaphragms.**

IV.4. Fresnel diffraction

IV.4.1. Diffraction by a hole along the axis

Let us look for the intensity of the wave diffracted by a circular pupil illuminated by a plane wave, and let us look more particularly at the diffracted field along the axis of the pupil. Consider an opaque screen pierced with a circular hole of diameter 2ailluminated by a monochromatic plane wave in normal incidence (Figure.IV.5).

The obstacle is placed in the z = 0 plane so that the wave state in the $z = 0^+$ plane is given by:

$$\psi(M) = \begin{cases} \psi_0 \ si\ \rho < a \\ 0 \quad is \quad non \end{cases}$$

In the absence of an obstacle, the incident wave has a uniform intensity$I_0 = \frac{1}{2}|\psi_0|^2$. Under the Huygens-Fresnel principle, the diffracted wave in M is written:

$$\psi(P) = \iint_S K\psi(M)\frac{e^{-ikr}}{r}dS = K\psi_0 \int_{\theta=0}^{2\pi}\int_{\rho=0}^{a} \frac{e^{-ik\sqrt{\rho^2+z^2}}}{\sqrt{\rho^2+z^2}}\rho d\rho d\theta$$

Where the polar system (ρ, θ) was chosen to locate the point P. The calculation of the double integral is divided into a product of two simple integrals (Fubini's theorem):

$$\psi(P) = K\psi_0 2\pi \int_{\rho=0}^{a} \frac{e^{-ik\sqrt{\rho^2+z^2}}}{\sqrt{\rho^2+z^2}} \rho d\rho = K\psi_0 2\pi \left[\frac{i}{k} e^{-ik\sqrt{\rho^2+z^2}}\right]_0^a$$

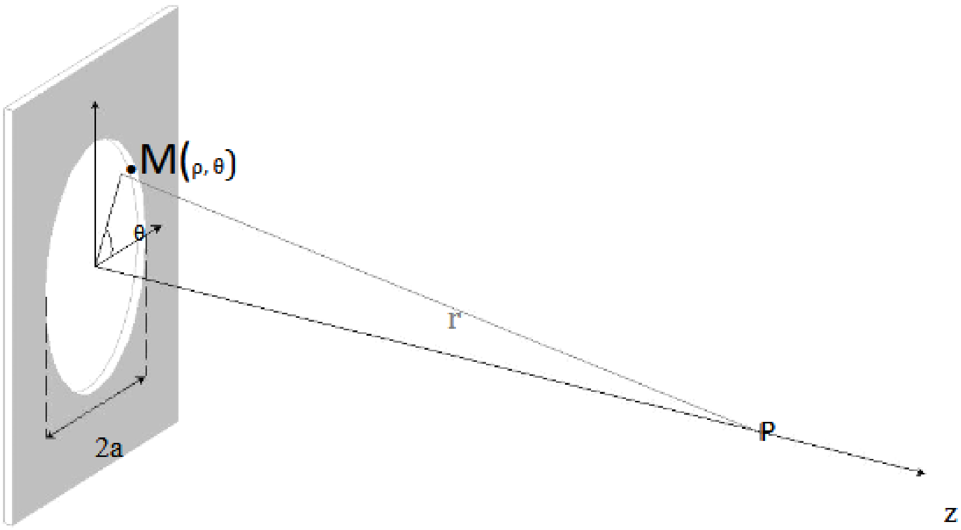


Figure IV.5: Parameterization of the problem diffraction by a hole along the axis

Finally, the complex amplitude of the diffracted field in M is:

$$\psi(P) = \frac{i2\pi K}{k}\left[\psi_0 e^{-ik\sqrt{a^2+z^2}} - \psi_0 e^{-ikz}\right]$$

If we use the value of the inclination factor K done by the Fresnel-Kirchhoff theory, or $K = \frac{i}{\lambda}$ here, we find

$$\psi(P) = \psi_0 e^{-ikz} - \psi_0 e^{-ik\sqrt{a^2+z^2}}$$

Result that can be interpreted simply: the first term corresponds to the incident wave (in $e^{-(\omega t - kz)}$), and the second to a wave from the edge of the hole, emitted with a delay of π (due to the presence of the sign -) and traveling an optical path equal to $\sqrt{a^2+z^2}$. An unexpected result is that the wave deflected by the edge presents in P the same amplitude as the incident wave. As a result, this overlay can produce completely destructive interference. Indeed, the calculation of light intensity in P gives:

$$I = \frac{1}{2}\psi\psi^* = 2I_0\left[1 - \cos(k\sqrt{z^2+a^2} - kz\right]$$

The graph of the intensity as a function of z (Figure IV.6) clearly indicates the existence of zero minima resulting from the destructive interference between the incident wave and the wave diffracted by the edges. The minima check the condition:

$$\sqrt{z^2 + a^2} - z = p\lambda \quad avec \;\; p \in N^*$$

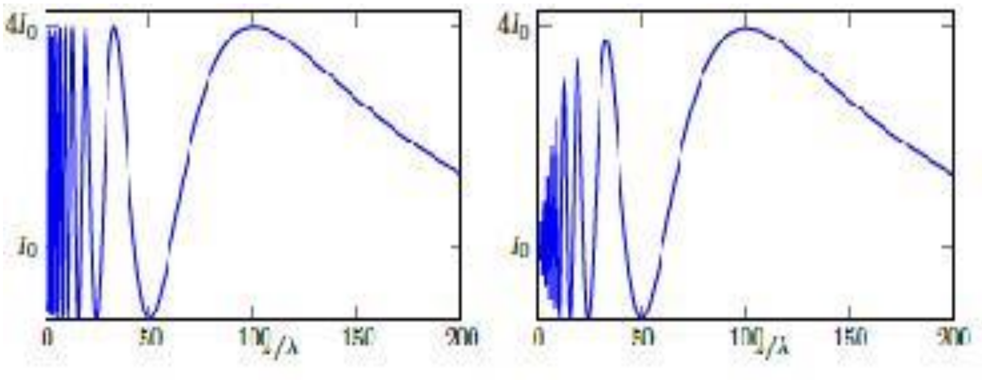


Figure IV.6: Intensity of light diffracted along the axis of a circular pupil of radius a = 10λ.

As can be seen, these intensity modulations appear when one is close to the diffracting obstacle and their number is of the order of a/λ. These rapid intensity modulations are characteristic of near-field diffraction. On the other hand, when

$$\sqrt{z^2 + a^2} - z < \frac{\lambda}{2} \;\; soit \;\; z > \frac{a^2 - (\lambda/2)^2}{\lambda}$$

The intensity luminous decreases of way monotonous until cancel out to the infinite. More precisely, the intensity decreases as $1/z^2$ lorsque $z >> \frac{a^2 - (\lambda/2)^2}{\lambda}$. This decrease in intensity along the axis is directly related to an angular dispersion of light energy; this is one of the characteristics of far-field diffraction, and is the subject of the next chapter. The previous calculation assumes K constant tilt factor. In reality, as we have already seen, the emission of spherical wavelets is not isotropic. When this effect is taken into account, the relationship

$$I = I_0 \left[1 + \frac{z^2}{z^2 + a^2} - \frac{2z}{\sqrt{z^2 + a^2}} \cos(k\sqrt{z^2 + a^2} - kz)\right]$$

We find the intensity modulations, but their amplitude decreases when $z \to 0$. Thus $I \to I_0$ when we approach the plane of the circular pupil, according to the hypothesis that the incident wave is not disturbed in the plane $z = 0$.

IV.4.2. Diffraction by a hole off the axis

The calculation of the field diffracted by a circular aperture in a point M located outside the axis presents more difficulties than the previous calculation (Figure IV.7). First, the cylindrical symmetry of the problem invites to locate the point P by its cylindrical coordinates (ρ', θ', z). As before, the point M is located by its polar coordinates (ρ, θ).

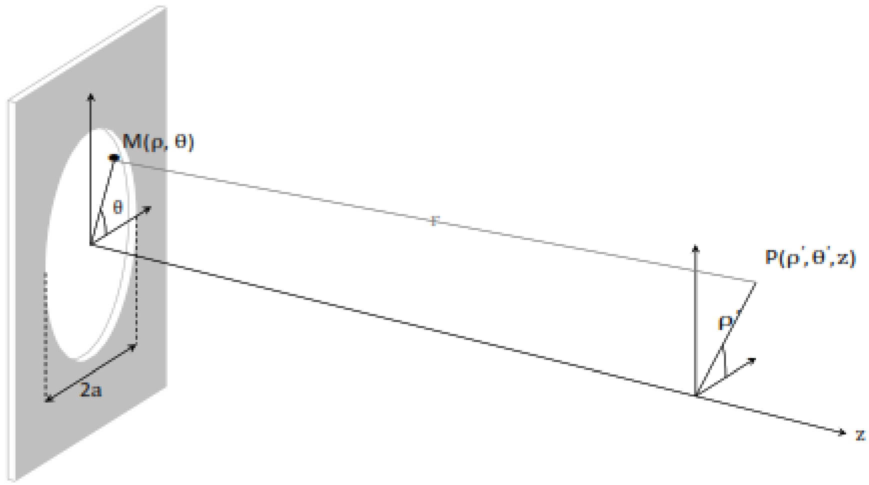


Figure IV.7: Parameterization of the problem of diffraction by a hole off the axis

According to the Fresnel formula, the diffracted field is written

$$\psi(P) = \iint_S K\psi(M)\frac{e^{-ikr}}{r}dS \qquad avec\, r = MP = (\rho^2 + Z^2 + \rho'^2 - 2\rho\rho'\cos(\theta' - \theta))^{1/2}$$

The Oz axis rotation invariance results in wave independence with θ' . Therefore, one can choose $\theta' = 0$. One gets

$$\psi(P)K\psi_0 \int_{\theta=0}^{2\pi}\int_{\rho=0}^{a} \frac{e^{-ik\sqrt{\rho^2+Z^2+\rho'^2-2\rho\rho'\cos\theta}}}{\sqrt{\rho^2 + Z^2 + \rho'^2 - 2\rho\rho'\cos\theta}}\rho d\rho d\theta$$

Let us rather focus on the properties of the diffracted field obtained by solving numerically the previous integral. The figure IV.8 shows the evolution of the intensity received by a sensor located at $z = 1000\lambda$ of the pupil whose radius varies between 10 and 90λ. The following observations can be made.

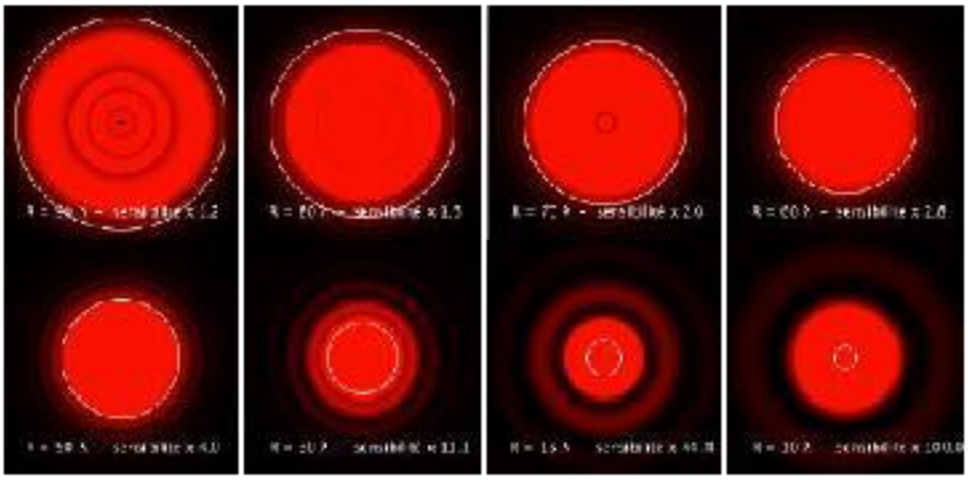


Figure IV.8: Intensity received on a sensor located at $z = 1000\lambda$ from a diffracting pupil of radius R (simulation). The sensitivity is fixed so the number of captured photons remains constant. The white circle indicates the edges of the geometric image.

- ❖ For large diameters, the spot is quite close to what the laws of geometric optics provide, except that dark rings appear.
- ❖ As the diameter decreases, the number of dark rings in the geometric image decreases. Note that the side dimension of the light spot is minimal when $z \approx a^2/\lambda$ (here for a radius of 30λ). Near field diffraction produces a slight focusing effect; effect consistent with the fact that the light intensity is maximum in the center when $z \approx a^2/\lambda$.
- ❖ Finally, a further decrease in the diameter of the pupil leads to an increase in the size of the luminous spot, which exceeds the size predicted by the laws of geometric optics. We will see in the next chapter that for small diffracting objects ($a << \sqrt{\lambda z}$) or, what is equivalent, for large observation distances ($z \approx a^2/\lambda$), the diffraction spot widens proportionally to zendragging at the same time a decrease in intensity in $1/z^2$.

IV.5. Infinité diffraction (Fraunhofer approximation)

Light a diffracting pupil with a plane wave in normal incidence, and then observe the diffracted light on a screen. From a certain distance, the diffraction pattern presents a unique pattern that simply expands as we move away from the observation screen. This type of diffraction is called Fraunhofer diffraction.

A plane wave in normal incidence arrives on a diffracting plane pupil (S) placed in $z = 0$. The distribution of light intensity is observed on a plane screen at the distance z obstacle. The notations are the same: $M(x, y, 0)$ locates a point of the diffracting pupil, and $P(\ x\ , y\ , z)$ at point of the observation screen (Figure IV.9).

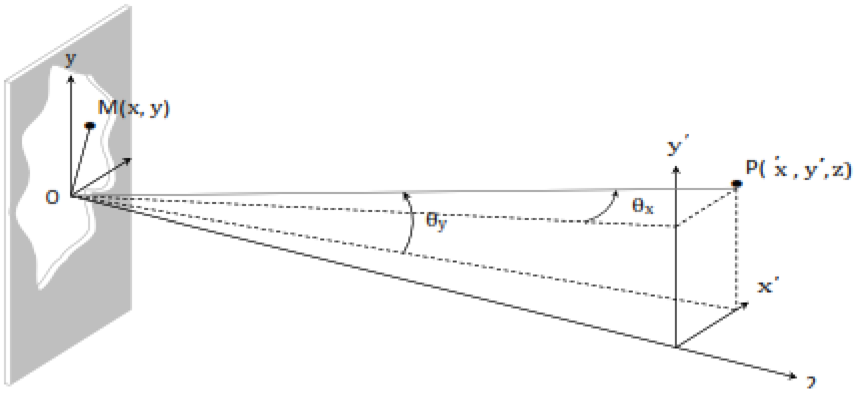


Figure IV.9: Setting up the far-field diffraction problem.

The Fraunhofer consisted approximation to be placed in the paraxial approximation $(x', y' << z)$and in the far field $(z >> x, y)$. Let us resume the expression of the diffracted wave predicted by the Huygens- Fresnel theory:

$$\psi(p) = \frac{i}{\lambda} \iint_S \psi(M) \frac{e^{-ikr}}{r} dxdy \qquad avec \quad r = PM$$

expression in which the obliquity factor is taken equal to 1 because the rays coming from M and arriving in P are little inclined with respect to the axis (Oz). Let's locate the point P using the two angles θ_x and θ_y que form (OP) with the axis (Oz):

$$\sin\theta_x = \frac{x'}{OP} \qquad and \qquad \sin\theta_y = \frac{y'}{OP}$$

As the observation is done at great distance, r varied little when M travels the integration domain, so that one can approach $1/r$ par $1/OP$. However, for the term of kr il phase must be more precise, because when M traverses the diffracting surface, it is enough that *the* r varied distance of $\lambda/2$ for the term e^{-ikr} change sign. We have

$$\vec{r} = \overrightarrow{MP} = \overrightarrow{OP} - \overrightarrow{OM} \qquad or \quad r^2 \simeq OP^2 - 2\overrightarrow{OP}.\overrightarrow{OM} + OM^2$$

Let's use the approximation$\sqrt{1-\varepsilon} \simeq 1 - \varepsilon/2$, then keep only the terms of order one in x et y. We end up with

$$r \simeq OP - \frac{\overrightarrow{OP}.\overrightarrow{OM}}{OP} = OP - x\sin\theta_x - y\sin\theta_y$$

The wave diffracted in far field is thus written

$$\psi(P) \simeq \frac{i}{\lambda}\frac{e^{-ik.OP}}{OP}\iint_S \psi(M)e^{ik(x\sin\theta_x - y\sin\theta_y)}dxdy$$

After this calculation, the light intensity is obtained by taking the square of the module of $\psi(M)$. In practice, we are interested in variations of intensity in the observation plane. Therefore, one is often content to calculate the integral of equation (5.1) since the multiplier term is constant in module $(1/OP \simeq 1\,/z)$:

$$I(P) \propto \left|\iint_S \psi(M)e^{ik(x\sin\theta_x - y\sin\theta_y)}dxdy\right|^2$$

This last relation shows that the distribution of the intensity depends on the position of M viauniquement the variables $sin\theta_x$ and $sin\theta_y$. In other words, we have

$$I(x', y', z) \propto f(x'/z\,, y'/z)$$

Which means that when z augmented of a certain factor (one moves back the sensor), the diffraction pattern undergoes a dilatation of this same factor. Finally, if we forget for a moment the fact that the intensity decreases as we move away from the diffracting pupil, the diffraction spot retains its shape to the dilation factor.

IV.5.1. Experimental Conditions

The simplest setup is to place a point source in the focus of a thin lens to form a flat wave. We then interpose the diffracting pupil to study, then diffracted light is collected on a screen placed far enough away. This is to clarify the latter point (Figure.IV.10).

Let us repeat the reasoning of the previous section:

$$\vec{r} = \overrightarrow{MP} = \overrightarrow{OP} - \overrightarrow{OM} = OP\sqrt{1 + \frac{OM^2 - 2\overrightarrow{OP}.\overrightarrow{OM}}{OP^2}}$$

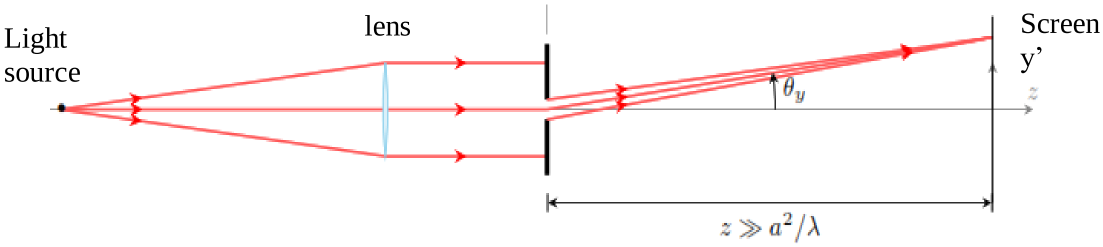


Figure IV.10: Observation of diffraction within the framework of the Fraunhofer approximation.

The Fraunhofer approximation consists in neglecting the influence of the quadratic term in,e^{ikr} which assumes

$$\vec{r} = \overrightarrow{MP} = \overrightarrow{OP} - \overrightarrow{OM} = OP\sqrt{1 + \frac{OM^2 - 2\overrightarrow{OP}.\overrightarrow{OM}}{OP^2}}$$

The Fraunhofer approximation consists in neglecting the influence of the quadratic term in,e^{ikr} which assumes

$$\frac{OM^2}{OP} << \lambda \quad \forall M \in (S)$$

In paraxial conditions we have OP ≃ z , and if we note the characteristic size of the diffracting pupil, the Fraunhofer approximation is valid provided that

$$z >> \frac{a^2}{\lambda}$$

Finally, the diffraction field can be separated into three zones, each zone corresponding to a different approximation level (Table IV.1 below).

$$z \gg \frac{a^2}{\lambda}$$

Pupil-to-screen distance	$z \sim \lambda$	$z \gg \lambda$	$z \gg \lambda$ et $z \gg \frac{a^2}{\lambda}$
Valid theory	Mawell	Huygens-Fresnel	Fraunhofer

Table IV.1: Different levels of approximation

In practice, the diffraction screen must be placed far enough away for the Fraunhofer approximation to be valid. In red light ($\lambda \sim 1 \mu m$), we find

$$for\ a = 1\ cm \to z >> 100m$$

$$for\ a = 1\ mm \to z >> 1m$$

$$for\ a = 1\ \mu m \to z >> 1\ cm$$

This constraint is accompanied by a decrease in brightness of the phenomenon. Indeed, according to the relation (5), the intensity in the center of the screen is given by

$$I(0,0,z) = \frac{A^2}{\lambda^2 z^2} I_0 << I_0$$

Where I_0 is the intensity of the incident light and A the area of the diffracting pupil. For example, for a ray hole at 1mm, one will choose 10m, and the intensity at the center of the screen will be of the order of I I0/100.The low brightness of the phenomenon often requires the use of a laser.

IV.5.2. Diffraction by a rectangular pupil

Consider a rectangular pupil of width $2a$ following (Oy) and 2b of length $2b$ suring (ox). Send a plane wave in the plane of the diffracting pupil. The state of vibratory is written:

$$\psi(M) = \begin{cases} \psi_0 & si\ \ |x| < a\ et\ |y| < b \\ 0 & if\ not \end{cases}$$

Where ψ_0 is constant amplitude? The wave diffracted to infinity in the direction given by θ_x and θ_y writes:

$$\psi(\theta_x, \theta_y) = \frac{i}{\lambda} \frac{\psi_0 e^{-ik.OP}}{OP} \int_{-b}^{b} e^{ik(y \sin\theta_y)} dy \int_{-a}^{a} e^{ik(x \sin\theta_x)} dx$$

Integral relationship

$$\int_{-x_0}^{x_0} e^{iax}\, dx = \left|\frac{1}{2a} e^{iax}\right|_{-x_0}^{x_0} = 2x_0 \operatorname{sinc}(ax_0)$$

Where the cardinal sinus function has been defined $sincx \triangleq \frac{sin\, x}{x}$. Thus, one gets

$$\psi(\theta_x,\theta_y)=\frac{i}{\lambda}\frac{\psi_0 4ab}{OP}e^{-ik.OP}\,\mathrm{sinc}\left(\frac{2\pi a\sin\theta_x}{\lambda}\right)\mathrm{sinc}\left(\frac{2\pi a\sin\theta_y}{\lambda}\right)$$

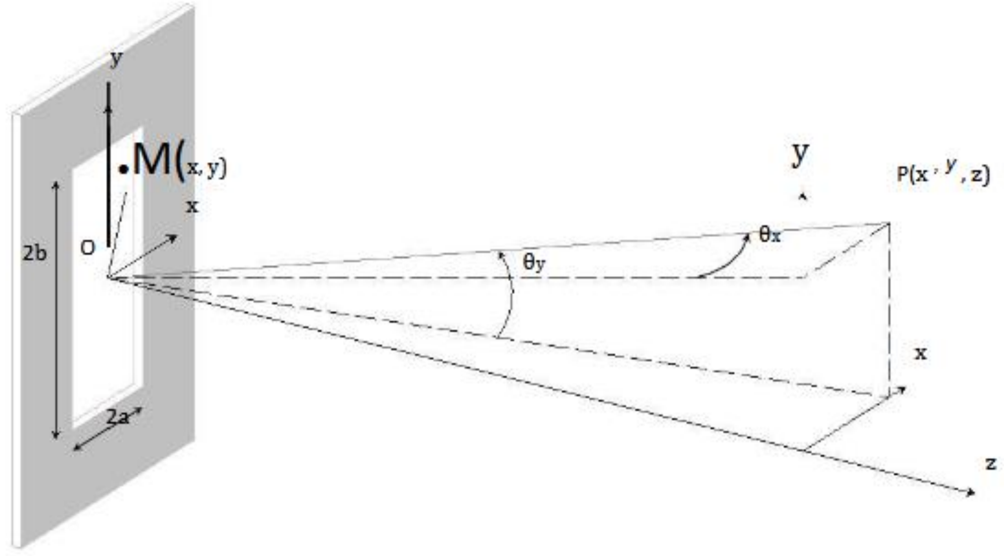


Figure IV.11: Diffraction by a rectangular pupil

And the light intensity is :

$$I(\theta_x,\theta_y)=I_{max}\left[\mathrm{sinc}\left(\frac{2\pi a\sin\theta_x}{\lambda}\right)\mathrm{sinc}\left(\frac{2\pi a\sin\theta_y}{\lambda}\right)\right]^2$$

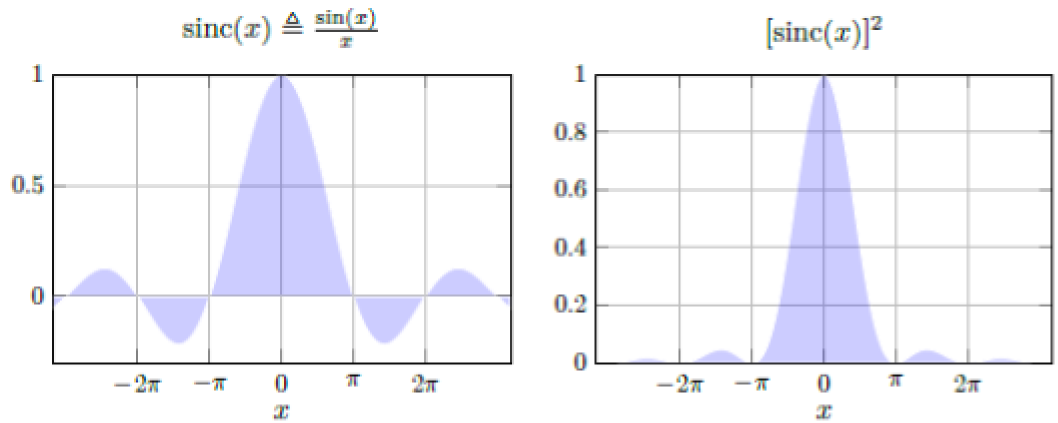


Figure VI.12: Graphs of the cardinal sinus function and its square.

With I_{max} light intensity along the optical axis ($\theta_x=\theta_y=0$). The figure IV.13 below shows the pattern collected on a screen located in the Fraunhofer area ($z\gg \max(a^2,b^2)/\lambda$) We can see that there are lines of zero intensity corresponding to the cancellation of one of the two cardinal sinuses. Note that the diffraction figure has a quadratic symmetry like the pupil. The elements of symmetry of the pupil are found in the diffraction figure as the Curie principle requires.

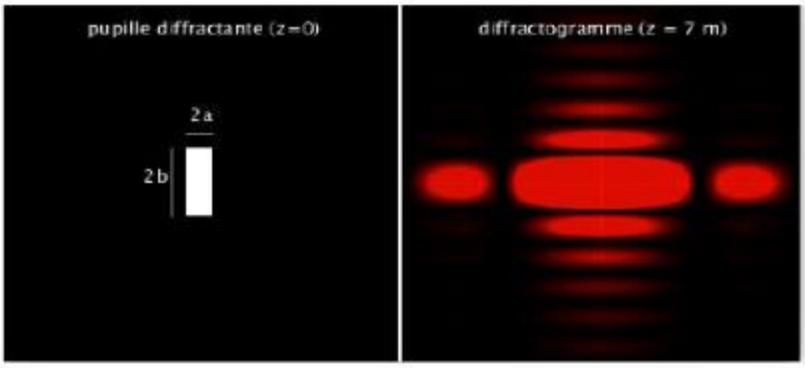


Figure VI.13: Diffracting pupil (left) and diffraction spot (right). Simulation performed for $\lambda = 700nm, a = 0.56\,mm, b = 3a\ and\ z = 7\,m.$

While geometric optics foresee a rectangular spot by virtue of the rectilinear propagation of light, taking into account diffraction reveals a central diffraction task much wider than the **geometric image. Indeed, the central spot is located in an angular space characterized by**

$$|\sin\theta_x| < \frac{\lambda}{2a} \quad and \quad |\sin\theta_y| < \frac{\lambda}{2a}$$

In the paraxial approximation, one can approach $\sin\theta_x$ and $\sin\theta_y$ by respectively x'/z and in y'/z. Also the main diffraction spot is localized the space zone defined by

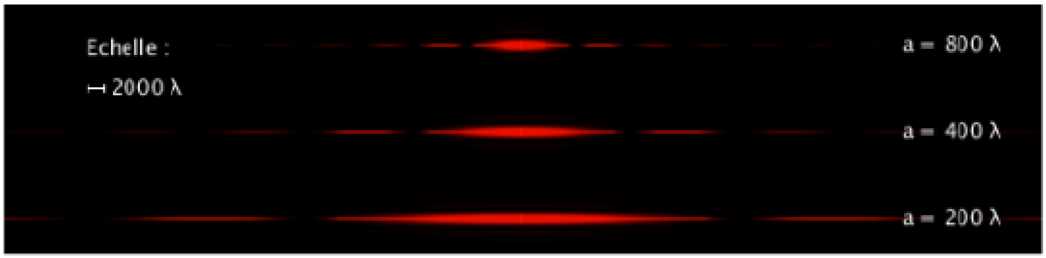
$$|x'| < a' = \frac{z\lambda}{2a} \qquad and \qquad |y'| < b' = \frac{z\lambda}{2b}$$

In the Fraunhofer area ($z \gg \max(a^2, b^2)/\lambda$), there is necessarily $a' \gg a$ and $b' \gg b$.

IV.5.3. Slit diffraction

The case of fine slit is obtained by making tender b to infinity. According to what we have seen on the rectangular pupil, we foresee a scattering of the light according to (Ox'), and almost not vertically. Let us therefore look at the distribution of the luminous intensity in $y = 0$ (Figure IV.14). The formula of intensity comes:

$$I(\theta_x, \theta_y = 0) = I_{max}\left[\text{sinc}\left(\frac{2\pi a \sin\theta_x}{\lambda}\right)\right]^2$$



FigureIV.14: Diffraction by a slot in the plane $z = 107\,\lambda$. The simulation shows the influence of the width on the diffractogram.

IV.6. Conceptual Questions / Exercises for Diffraction

1. **Concept:** Why does light spread out after passing through a narrow slit? Explain using the Huygens-Fresnel principle.
2. **Concept:** What is the difference between Fresnel and Fraunhofer diffraction? Under what conditions is the Fraunhofer approximation valid?
3. **Single Slit Minima:** Light with a wavelength of 550 nm is incident on a single slit. The first diffraction minimum (m=1) is observed at an angle of 5.0°. What is the full width (W) of the slit?
4. **Single Slit Central Maximum:** Light of wavelength 600 nm passes through a single slit of full width 0.10 mm. The diffraction pattern is observed on a screen 2.0 *m* away. Calculate the linear width of the central bright maximum on the screen.
5. **Rectangular Aperture:** Describe the general appearance of the Fraunhofer diffraction pattern produced by a rectangular aperture. How does the pattern change if the aperture is made narrower in the x-direction but taller in the y-direction?
6. **Fresnel vs. Fraunhofer Condition:** A circular aperture has a diameter of 0.5 *mm*. If illuminated by light of wavelength 500 nm, estimate the minimum distance z to the screen for the Fraunhofer approximation to be considered valid ($z \gg a^2/\lambda$, where a is radius here).
7. **Significance:** Why is diffraction a fundamental limitation on the resolution of optical instruments like telescopes?

8. **Rectangular Aperture - Zero Intensity:** A rectangular aperture has full dimensions w_x = 0.20 mm (width) and w_y = 0.30 mm (height). It is illuminated by light of wavelength λ = 500 nm. Find the angles θ_x and θ_y that define the location of the *first* vertical line of zero intensity and the *first* horizontal line of zero intensity in the Fraunhofer pattern (excluding the axes themselves).

9. **Square Aperture:** Describe the Fraunhofer diffraction pattern produced by a square aperture (width = height = *w*). How does it relate to the pattern from a rectangular aperture?

10. **Fresnel Diffraction - Axial Intensity:** Consider Fresnel diffraction from a circular aperture of radius a illuminated by wavelength λ. According to the simple interference model between the direct wave and the edge wave , minima in intensity occur along the axis at distances z satisfying $\sqrt{(z^2 + a^2)}$ - z = pλ where p is a positive integer. If a = 1.0 mm and λ = 500 nm, find the approximate distance z for the first axial minimum (p=1). (You might need to solve for z, perhaps assuming z >> a initially or rearranging the equation).

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Online Resources:

1. **HyperPhysics (Georgia State University)**
   - *Website:* http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/diffrac.html
2. **PhET Interactive Simulations (University of Colorado Boulder)**
   - *Website:*https://phet.colorado.edu/en/simulations/filter?subjects=physics&topics=light-and-radiation
3. **OSA Publishing** (Optica)
   - *Website*: https://opg.optica.org
4. **MIT OpenCourseWare (Physics III: Vibrations and Waves)**
   - *Website:* https://ocw.mit.edu/courses/8-03sc-physics-iii-vibrations-and-waves-fall-2016/